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Practices of Textual Changes in Medieval Mathematical Manuscripts: The Case Study of the Division of Geometric Figures in *Ḥibbur ha-Meshiḥah ve-ha-Tishboret* and *Liber embadorum*

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Article

Practices of Textual Changes in Medieval Mathematical Manuscripts: The Case Study of the Division of Geometric Figures in *Ḥibbur ha-Meshiḥah ve-ha-Tishboret* and *Liber embadorum*

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Abstract

One of the well-known medieval mathematical treatises written in Hebrew is Rabbi Abraham bar Ḥiyya's *Ḥibbur ha-Meshiḥah ve-ha-Tishboret*, composed between 1116 and 1145. The *Ḥibbur* survives in (at least) eight manuscript copies, produced between the fourteenth and the sixteenth centuries, as well as in a Latin translation titled *Liber embadorum*, made in 1145 by Plato of Tivoli. Of the latter, five extant copies from the fifteenth and the sixteenth centuries are known. While the Latin translation adheres to the general structure of the *Ḥibbur*, it also exhibits specific changes that are indicative of the historical development of both the Hebrew and the Latin texts. By focusing exclusively on the extant manuscripts – rather than on any endeavor to (re)construct a history of the *Urtext*, whether in Hebrew or Latin – one can observe not only various omissions, comments, and notes added to the various copies, but also that such changes modify the character of the text, rendering it less or more practice- or theory-oriented.

What do these additions and omissions reveal about how mathematical texts were regarded in the fourteenth to sixteenth centuries? Can we draw conclusions about how these texts were read, copied and used? To answer these questions, this paper examines three sections of the *Ḥibbur* and *Liber embadorum*, all of which concern the study of the division of geometric figures. I argue that a refined typology of the additions, omissions, added notes, comments and diagrams can help us to see these manuscripts not as 'copies' of an 'original text', but as complex mathematical written artefacts in their own right, each with unique characteristics.

Keywords

Ḥibbur ha-Meshiḥah ve-ha-Tishboret, *Liber embadorum*, medieval mathematical manuscripts, textual fluidity, degrees of solvability, Hebrew manuscripts, history of mathematics

1. Introduction: Fluidity, (para)content and textual changes in mathematical medieval Hebrew manuscripts

One of the well-known medieval mathematical works in Hebrew is the treatise of Rabbi Abraham bar Ḥiyya (*c.*1065–1145; hereafter: AbH): *Ḥibbur ha-Meshiḥah ve-ha-Tishboret* (חיבור המשיחה והתשבורת; hereafter: *Ḥibbur*), composed between 1116 and 1145. The title may be translated as 'A Treatise on Measuring Areas and Volumes'. AbH, who was titled in Hebrew 'ha-Nasi' (honorary leader) and in Latin 'Savasorda' ('Head of the guard'),[1] had a unique accomplishment: the development of an entire mathematical terminology in Hebrew for the first time. The *Ḥibbur* is also recognized for being the first Hebrew work to present the Islamic techniques for solving problems involving the square of an unknown variable through geometric means.[2] The treatise is composed of four parts (*she'arim*, שערים), which I refer to as Books I–IV. The *Ḥibbur* was translated into Latin as *Liber embadorum* in 1145 by Plato of Tivoli, and it is possible that Plato of Tivoli and AbH have cooperated while the former was preparing the translation. This cooperation may indicate that the Hebrew and the Latin texts are almost contemporary.[3] *Liber embadorum* may also have influenced *De practica geometrie* of Leonardo Fibonacci (*c.*1170–1240).[4] Both the *Ḥibbur* and *Liber embadorum* were copied several times over the centuries. Eight Hebrew manuscripts are known to contain the complete text (or significant sections) of the *Ḥibbur*, and five Latin manuscripts known to contain the complete text (or significant sections) of *Liber embadorum*.

Starting in the mid-nineteenth century, several attempts were made to produce a critical edition of the *Ḥibbur*, comparing and presenting the differences among the various Hebrew manuscripts. Moritz Steinschneider published a partial critical edition in 1893,[5] but the most well-known critical edition was published by Michael Guttmann in two parts in 1912 and 1913.[6] This edition was based on former, partial critical editions of the *Ḥibbur* that Guttmann had prepared and published during the previous decade. In his edition, Guttmann numbered the various sections of the work, and while this numbering does not appear in any of the Hebrew manuscripts, I will nevertheless use it to refer to the different sections.[7] Ten years prior to Guttmann's publication of the first part of his critical edition, Maximillian Curtze had compared two Latin manuscripts in 1902, transcribed them, and translated the Latin transcription into German. His comparison culminated in a bilingual text of *Liber embadorum* based on these two manuscripts.[8] Curtze also numbered the sections in his edition – a numbering I will also adopt.[9] While the two manuscripts with which Curtze worked contained only minor textual variations (or micro-changes; see below) when compared to each other, this was not the case for the manuscripts with which Steinschneider and Guttmann were working.

[1] 'Savasorda' is a transcription of the Arabic title *sâhib al-shurtah*.

[2] Obviously, AbH did not present variables or equations, and the above formulation is a modern one to describe the *Ḥibbur*'s mathematical content.

[3] For supporting evidence for this hypothesis, see Lévy 2001, 38.

[4] Hughes 2008, xxiii–xxiv. The extent to which Fibonacci was influenced by *Liber embadorum* is not clear, and it seems that *Liber embadorum* was not the main source of Fibonacci's *De practica geometrie*, which was, rather, Abu Bakr's *Liber mensurationum*. See Høyrup 2019.

[5] Steinschneider 1893.

[6] I abbreviate this edition as HMT.

[7] For example, when referring to section [35] in the *Ḥibbur* it therefore refers to Guttmann's numbering.

[8] I refer to the critical edition made by Curtze as Curtze. Guttmann knew of Curtze's edition, as he mentions it in HMT, xxxii. However, apart from mentioning it, he does not take it into account in any way. Curtze does not mention Guttmann at all, though this is hardly surprising: the first partial critical edition that Guttmann published was in 1903. However, he does mention Steinschneider 1896, which discusses, among other subjects, the works of AbH (Curtze, 5).

[9] In contrast to Guttmann, Curtze starts the numbering anew at the beginning of every Book.

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The numerous textual differences and variants between the Hebrew manuscripts led Steinschneider to hypothesize that two different versions of the *Ḥibbur* may have been composed by AbH from the outset: one longer version that was more 'mathematical' or theory-oriented, sometimes including proofs for the geometrical propositions, and another shorter and more 'practical' version, devoid of these proofs.[10] Guttmann rejected this hypothesis, asserting that only one *Urtext* existed, and that the textual changes in the various manuscripts arose from scribal omissions. Based on this assumption, Guttmann published an 'imaginary' *Urtext* in 1912–1913 – an eclectic reconstruction that quotes from various manuscripts when necessary, but which, as a whole, cannot be found in any extant witness. Furthermore, Guttmann neither prepared a stemma nor elaborated on the possible reasons for the various textual differences between the manuscripts. He simply determined, for example, that there was one manuscript which included all the proofs, whereas the other manuscripts merely omitted several of them.[11] The hypothesis concerning the existence of a 'single' version of the *Ḥibbur* may indeed be correct, but as will be demonstrated below, one may also suggest that AbH's original version was in fact the shorter one, and that the other versions represent later elaborations on this version. Nonetheless, Guttmann's method of reconstructing such an *Urtext* deserves criticism. As has recently been demonstrated, such reconstructions and, more broadly, any analysis of these copies within the framework of one or two *Urtext(s)* are no longer reasonable,[12] as they fail to reflect the fluidity of the texts in the Hebrew manuscripts or the changes between them.[13] This textual fluidity, including the macro- and micro-changes between the different versions, will be discussed in the current paper, focusing on various sections of Book III.

It is important to stress here that such phenomena are not unique. Peter Schäfer, Israel Ta-Shema, Malachi Beit-Arié, as well as Gad Freudenthal and Jean-Marc Mandosio, all emphasize the textual fluidity of medieval Hebrew works.[14] Ta-Shema observes, for example, that such works 'often seem perplexingly left *in mediis rebus* and intrinsically incomplete', highlighting several types of corrections, editing methods and additions.[15] Beit-Arié, to give another example, emphasized the 'dynamic state of the text and its transformational character', and claimed that the scribes of medieval Hebrew manuscripts 'certainly did not view copying as mechanical reproduction, but as a critical editorial operation involving emendation, diagnostic conjecture, collation of different exemplars and even the incorporation of external relevant material and the copyist's own opinion'.[16] Freudenthal and Mandosio, while characterising lapidary texts as 'texts in flux', assert

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10 Steinschneider 1867, 17–18.

11 HMT, xxx. In fact, one recognizes two manuscripts which can be considered as the 'longer' or 'mathematical' ones (see p. 67), but Guttmann did not have in his possession the second manuscript in its entirety. Here one must recall that Guttmann compared five manuscripts, but was able to see only three of them, while he had only obtained copies of the other two.

12 It should be stressed that such hypotheses did not exist for *Liber embadorum*.

13 As an attempt to solve such imaginary reconstructions and to account for the various changes, Friedman and Garber 2023, 170–172 suggested to present all eight Hebrew manuscripts side-by-side as a digital edition. Cf. Schäfer 1981.

14 Schäfer 1986; Beit-Arié 1993; Ta-Shema 1993; Freudenthal and Mandosio 2014.

15 Ta-Shema 1993, 17, 22.

16 Beit-Arié 1993, 34, 50.

that medieval Hebrew 'scientific or philosophical texts […] were usually copied by more or less professional copyists or scribes whose goal was a faithful reproduction of their author's text';[17] however, one must ask whether such a statement is valid for all 'scientific' texts. Indeed, given the textual fluidity observed in our case study, the analysis of the various copies of the *Ḥibbur* raises the question of whether medieval Hebrew *mathematical* texts possess additional characteristics that either limit or expand the scope of such textual fluidity and textual change.

To examine these issues more closely, it is helpful to differentiate between macro- and micro-changes. Macro-changes are more extensive changes that affect the overall structure of the text, whereas micro-changes refer to more local and restricted alterations.[18] In this paper I would like to further refine this typology by considering another type of change: changes which render the text scholarly and theoretical or, conversely, more practical. Such additions (or omissions) attest not only to the fluidity of the text, but also to how some scribes (though certainly not all) actively attempted to (re)shape it. These scribal interventions can result in direct changes of the text or in the addition, alteration, or omission of paracontent; by paracontent I follow here the CSMC definition of paracontent, namely, 'a set of visual signs (writing, images, marks) that is present in a manuscript in addition to the core-content'.[19] Its function is, in part, to structure, comment on and document the content. In the case of the *Ḥibbur* and *Liber embadorum*, one may classify the scribes' comments and colophons as paracontent, as well as other elements such as introductions to the various Books. Diagrams may also be considered as paracontent, but in this paper they are a borderline case: diagrams can be regarded as an essential component of a geometrical exercise, hence belonging to the core content. Moreover, the introductions, even though usually not containing any mathematical content, occasionally provide essential justifications for the necessity of engaging with the various exercises. Such an examination will necessitate a close reading of the manuscripts, and for this reason I concentrate only on three sections from the *Ḥibbur* and *Liber embadorum*.

The focus on paracontent, on micro vs. macro-changes, and on more scholarly vs. more practical additions would facilitate a more thorough investigation of the various textual changes between the manuscripts, including the presence or absence of diagrams, the addition of notes in the margins, or the inclusion of appendices to the *Ḥibbur* or *Liber embadorum*. Such omissions or additions, serving, for instance, to either critique solutions or to propose alternative approaches to mathematical practices, invite a reconsideration of the notion of solvability of mathematical problems in terms of degrees of solvability. As I will demonstrate in this paper, these degrees also indicate whether the manuscript's content leans towards a more practical or a more scholarly-theoretical nature. The term 'degrees of solvability' is employed to denote the various types of solutions to (mathematical) problems presented in the manuscripts. A geometric problem can be solved only by a specific numerical example that demonstrates its solvability, and such an example may even consist of

[17] Freudenthal and Mandosio 2014, 26.

[18] See Friedman and Garber 2023.

[19] Ciotti et al. 2018, 1.

only an approximation. However, a stronger, more rigorous degree of solvability may be detected for such a problem: a geometric proof can be provided, ensuring certainty. The proof may further be accompanied by a diagram that visualizes some of the procedures involved. These practices demonstrate the varying degrees of solvability that can differ from manuscript to manuscript.

To elucidate these issues, the paper will begin in Section 2 by introducing the *Ḥibbur* and *Liber embadorum*, as well as their known copies. Section 3 focuses on Book III of these manuscripts, which deals with the division of figures. This Book constitutes early textual evidence of Euclid's lost book *On Divisions* and is therefore of particular importance to the history of mathematics. Accordingly, I will concentrate on the first three sections of Book III, examining changes according to the typology introduced above, that is, examining the various macro- and micro-changes, such as the additions and omissions of diagrams, additions of structuring remarks by the scribes, or omission of mathematical explanations. In the subsequent section (Section 4), a more extensive examination will be conducted, focusing on one specific manuscript copy of *Liber embadorum*: the manuscript copied and annotated by Francesco Barozzi, a well-known sixteenth-century mathematician. The concluding section, Section 5, will then take a step back and discuss the nature of the various mathematical manuscripts discussed in this paper.

2. On the *Ḥibbur* and *Liber embadorum*

Before examining various sections of the *Ḥibbur* and of *Liber embadorum* in detail, I would like to outline the structure of these treatises as it appears in most of the manuscript copies (see below). Six of the eight Hebrew manuscripts, as well as the Latin manuscripts, share a common basic structure.[20] The remaining two Hebrew manuscripts represent abbreviated and reorganized versions of the treatise, epitomizing the material found in an earlier manuscript version of the *Ḥibbur*. In all but these two manuscripts, the *Ḥibbur* and its Latin translation consist of a general introduction, followed by four books, and end with a conclusion. Book I is more introductory and theoretical, whereas Books II–IV are more practical, dealing with the measurement and division of shapes. Book I provides geometrical and arithmetical definitions and, starting from section [27] onward, presents basic geometrical propositions, including a few on the similarity and congruence of polygons. As mentioned above, while some of the Hebrew manuscripts contain proofs for these propositions, others do not. This difference led Steinschneider to hypothesize the existence of two distinct versions.[21] Book II is subdivided into five parts. It addresses the measurement of quadrilaterals and triangles and contains the geometrical treatment of what are today termed quadratic problems. It also discusses the measurement of circles and their areas, as well as presenting a table for calculating the lengths of the arcs of a circle given the corresponding chords. Book III, which treats the division of the shapes previously examined in Book II into other figures, is based on Euclid's lost book on the division of figures. The last book, Book IV, provides a concise overview of three-dimensional solids. Finally, in some (but not all) manuscripts, an

[20] See also Lévy 2001; Friedman and Garber 2023.

[21] See Friedman and Garber 2023, 138–140.

'appendix' follows Book IV of both the *Ḥibbur* and *Liber embadorum*. The appendix contains additional exercises and proofs; however, the appendices differ in the *Ḥibbur* and *Liber embadorum*. Moreover, it is unclear whether these appendices were written by AbH or not. In some of them, AbH's name is explicitly mentioned, suggesting that this mention was added by the scribe in a later stage (see also p. 67).

Currently, eight Hebrew copies of the *Ḥibbur* are known to exist. These are listed below in alphabetical order according to their present location.[22] All of these manuscripts contain other texts alongside the *Ḥibbur*. The sigla used to refer to the manuscripts throughout the paper are indicated at the beginning of each reference. They are composed from the first letter of the library's name followed by the inventory number, if needed.

1. M256: Munich, Bayerische Staatsbibliothek, Cod. hebr. 256, fols 54^{r}–118^{r} (fourteenth or fifteenth centuries, Sephardic writing, unknown provenance).
2. M299: Munich, Bayerische Staatsbibliothek, Cod. hebr. 299, fols 45^{r}–103^{v} (fourteenth or fifteenth centuries, Sephardic writing, unknown provenance).
3. P995: Paris, Bibliothèque nationale de France, fonds héb. 995, fols 269^{r}–296^{v} (sixteenth century, Italian writing, unknown provenance).
4. P1048: Paris, Bibliothèque nationale de France, fonds héb. 1048, fols 1^{v}–64^{v} (fifteenth century, Italian writing, unknown provenance).
5. P1050: Paris, Bibliothèque nationale de France, fonds héb. 1050, fols 30^{r}–46^{r} (fourteenth or fifteenth centuries, Italian writing, unknown provenance).
6. P1061: Paris, Bibliothèque nationale de France, fonds héb. 1061, fols 22^{r}–105^{v} (1464, Byzantine writing, unknown provenance).
7. Pr2635: Parma, Biblioteca Palatina, Cod. parm. 2635 (De Rossi 1170), fols 6^{r}–60^{r} (1437, Sephardic writing, Lecce, Italy).[23]
8. V: Vatican City, Biblioteca Apostolica Vaticana, Vat. ebr. 400, fols 19^{r}–60^{v} (fourteenth or fifteenth centuries, Sephardic writing, unknown provenance).

It is important to note several peculiarities in the content of the witnesses listed above. First, manuscripts P995 and P1050 only contain parts of Books II–IV, lacking titles, and the copied sections are substantially reordered. For instance, parts of Book III appear both before and after parts of Book II. Given the absence of Book I, the text in these manuscripts can be considered as having a more practical character, likely intended for teaching. This interpretation is further substantiated by the fact that P995 and P1050 do not include the proofs for various propositions. Apparently, these manuscripts are copies of a manuscript that contained a later, reorganized version of the *Ḥibbur*. Second, manuscripts P1048, P1061 and V contain the aforementioned appendix, though V presents an expanded version of it compared to P1048 and P1061. The appendix in V

[22] In Wagner 2016, 296–313, various sections from the introduction, Book I, II and III of the *Ḥibbur* were translated into English. However, the translation was done from the 1912–1913 edition of Guttmann and should be accordingly revised.

[23] This manuscript contains only copies of texts of AbH: first (fols 1^{r}–6^{r}), the introduction of *Yesodei ha-Tevunah u-Migdal ha-Emunah* ('The Foundations of Wisdom and the Tower of Faith'), and afterwards the *Ḥibbur*.

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explicitly mentions 'ha-Nasi', which was, to recall, one of AbH's titles, and the *Ḥibbur* (referred to as ספר התשברת, 'A Treatise on Measuring Areas').[24] The content of the appendix in V (fols 61r–66r) addresses various geometric problems and propositions dealing with triangles, quadrilaterals, and chords and arcs of a circle. This was not the only addition in manuscript V: in Book II, section [52], the scribe also refers to 'ha-Nasi', which indicates a later addition of the scribe.[25] Third, manuscripts M256 and Pr2635 omit the conclusion that typically follows Book IV.[26]

What are the stemmatic relations among the various manuscripts? If one excludes P995 and P1050, which lack Book I, then, following Friedman and Garber,[27] it is evident that the presence and absence of proofs and drawings in Book I for the propositions in sections [30]–[41] are indicative of Steinschneider's hypothesis of two distinct versions. M256 and Pr2635 contain the proofs of all of these propositions, whereas the other four manuscripts do not. Therefore, M256 and Pr2635 may be considered copies of a longer (i.e. more 'mathematical', or more precisely, more 'theory-oriented') version of the *Ḥibbur*, whereas conversely, the other four are copies of a shorter version. Nevertheless, a more careful examination reveals that the four manuscripts representing the 'shorter' version are not uniform. P1048 and P1061 are almost identical; they (along with M256 and Pr2635) contain proofs for the propositions stated in sections [27]–[29] that are absent in manuscripts M299 and V. In this sense, manuscripts M299 and V are the 'shortest' manuscripts. Moreover, manuscript M299 has a few additional figures in Book I that do not appear in manuscript V. Manuscript V, in turn, as noted above, contains additions which do not appear in M299. Examining other Books of the *Ḥibbur*, it is evident not only that even manuscripts P1048 and P1061 exhibit some differences but also that M299 appears to be a copy of an earlier version of the *Ḥibbur* compared to the other manuscripts.[28]

As for *Liber embadorum*, although the Latin translation follows the general structure of the *Ḥibbur*, there are several differences, which have already been noted by Tony Lévy. In addition to the omission of Jewish textual elements such as references to rabbinical sources, both explicit and non-explicit citations or verses, as well as the introduction and the conclusion, there are numerous substantial differences in Book I.[29] For example, the Latin text (in all known copies) includes a list of postulates and common notions that is absent from the Hebrew manuscripts.[30] Lévy suggests that the Latin translation of the propositions in Book I of the *Ḥibbur* was probably derived from an Arabic version of Euclid's *Elements*.[31] Some of these differences may be explained by the fact

24 See V, fols 61v and 62v. It is essential to take note that the appendix was most probably written by one of the later scribes of the *Ḥibbur*.

25 See HMT, 33.

26 These macro-changes, among others, were elaborated in Friedman and Garber 2023, 137–154.

27 Friedman and Garber 2023, 138–140.

28 Friedman and Garber 2023, 156–157.

29 Lévy 2001, 53–55.

30 Curtze, 14 and 16.

31 See Lévy 2001, 54. Moreover, the list of postulates and common notions contains different terminology, and includes an additional axiom, absent from all the other Arabic-Latin versions of Euclid's *Elements* at that time. This, Lévy stresses, offers us an opportunity to understand the medieval history of *Euclides Latinus*. See also Rommevaux et al. 2001.

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that AbH was coining mathematical terminology in Hebrew for the first time, making it necessary to explain these terms and provide the linguistic and biblical background. Those explanations were, of course, unnecessary in the Latin translation. Another major difference is that the Latin manuscripts lack the extended proofs for the propositions in Book I that appear in the 'longer' version of the *Ḥibbur* (i.e. propositions [27]–[41]). This indicates that *Liber embadorum* was probably translated from a 'shorter' version of the *Ḥibbur*. Furthermore, Book III of the Latin manuscripts contains an additional proof for one of the statements, which does not appear in any of the Hebrew manuscripts.[32] This may indicate that Plato of Tivoli (perhaps collaborating with AbH) had access to other mathematical texts while translating the *Ḥibbur*, or to a redaction of the *Ḥibbur* that is not preserved in any of the surviving Hebrew manuscripts. This finding may point to a complex relationship between AbH and Plato of Tivoli, since one may conjecture that Plato of Tivoli may have consulted AbH during the preparation of the translation.[33]

There are five known copies of the Latin translation. I have ordered them alphabetically according to their current location. Three of the five manuscripts listed below contain also other texts besides the *Liber embadorum*. As above, the sigla of the manuscripts, which I use throughout the paper, are indicated at the beginning of each reference and are composed from the first letter of the library's name followed by the inventory number.

1. D390: Dublin, Trinity College Library, Mediaeval and Renaissance Latin Manuscripts, MS 390, fols 2^{r}–53^{v} (1565).[34]
2. F184: Florence, Biblioteca Medicea Laurenziana, S. Marco 184, fols 120^{r}–164^{v} (fifteenth century).
3. F36: Florence, Biblioteca Nazionale Centrale, Conv. soppr. J. VI. 36, fols 23^{r}–40^{v} (formerly S. Marco 207) (thirteenth to fifteenth centuries).
4. P7224: Paris, Bibliothèque nationale de France, lat. 7224 (the entire manuscript) (sixteenth century).[35]
5. P11246: Paris, Bibliothèque nationale de France, lat. 11246, fols 1^{v}–37^{r} (fifteenth century).[36]

A comprehensive investigation into the stemmatic relationships among the various manuscripts does not yet exist. Nonetheless, the analysis conducted in this paper concerning the sections in Book III reveals no significant differences, though some manuscripts lack diagrams or contain 'appendices' (see below). As previously mentioned, when examining the propositions in section [27]–[41] of Book I, it is evident that the Latin manuscripts do not contain any proofs. Thus, the Latin translation appears to have been prepared on the basis of a version of the *Ḥibbur* closer to that preserved in M299 and

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[32] See footnote 55.

[33] See Lévy 2001, 38.

[34] This manuscript contains only the copy of *Liber embadorum* (as well as an appendix). The digitised manuscript can be found here: <https://doi.org/10.48495/n583z200g> (last accessed on 1 October 2025).

[35] P7224 was one of the two manuscripts used by Curtze and the editor referred to it with the siglum B in his edition.

[36] P11246 is the second manuscript used by Curtze and the editor referred to it with the siglum A in his edition. Curtze notes that the images drawn in his edition are taken from this manuscript (Curtze, 5).

V. Additional supporting evidence for such a hypothesis will be presented in the following sections. Upon closer examination of manuscript D390, one notes that it contains extensive marginal notes, textual additions by the scribe, diagrams, and numerical calculations, none of which appear in any of the other manuscripts. This is not unexpected: the scribe of the text was Francesco Barozzi (1537–1604), a Venetian mathematician, astronomer and humanist, who was known for his various translations of Greek mathematical texts into Latin.[37] Indeed, Barozzi also translated Proclus' commentary of Euclid's *Elements* for the first time, adding various comments and notes (though only to Euclid's first book); this translation was published as *Procli Diadochi Lycii in primum Euclidis elementorum librum commentariorum* in Venice in 1560.[38] Additionally, he translated other Greek works, including those by Heron, Pappus, and Archimedes. The title page of D390 is revealing in this regard. As a note written by Barozzi himself makes it clear that he was very much aware of the nature of his textual interventions and his corrections:

> The *Book of the Lands* [*Liber de areis*] was composed in Hebrew by Savasorda the Jew and translated into Latin by Plato of Tivoli, cleared of infinite errors by Francesco Barozzi, explained with scholia and notes, and now published for the first time. In addition, Ersemidis's book *Liber de dimensionibus* corrected by the same Francesco Barozzi.[39]

It is noteworthy that the expression 'published for the first time' signifies that this text was intended for printing, indicating that Barozzi considered it the final version. However, such an edition was never published.

Barozzi's additions sometimes belong to the level of the core content, expanding what is found in *Liber embadorum*, while other elements fall into the category of paracontent (see Fig. 1). Although it is clear that the entire manuscript was written by Barozzi in one go, he himself establishes a physical separation between the 'original' text (the *Liber embadorum*) and his own additions, which are written in the margins. The presence of additions and comments by Barozzi's hand is not surprising, since he did so in other texts that he edited. Moreover, these annotations demonstrate the significance that Barozzi attributed to both the *Liber embadorum* and the *Ḥibbur*. Furthermore, an appendix was added to D390: fols 55ʳ–58ᵛ contain supplementary material consisting of geometrical problems, propositions, and their proofs. However, a more detailed examination of this material would go beyond the scope of the present paper.

In contrast to D390, the other manuscripts of the *Liber embadorum* contain almost no comments or additions from the anonymous scribes who copied them. F184 contains neither diagrams nor remarks from the scribe. Most importantly, it lacks the entire conclusion (found after Book IV in the other witnesses). Furthermore, fols 159ʳ–164ᵛ of this manuscript can be regarded as an 'appendix',

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37 See O'Connor and Robertson 2006.

38 See Proclus 1560. A translation into English of Proclus' work is in Proclus 1970.

39 D390, fol. 1ʳ: *Liber de areis a Savasorda Iudeo in Hebraico compositus et a Platone Tiburtino in Latinum sermonem translatus, a Francisco Barocio infinitis erroribus expurgatus atque scholiis et annotationibus illustratus ac nunc primum in lucem editus. Item Ersemidis liber de dimensionibus ab eodem Francisco Barocio restauratus.*

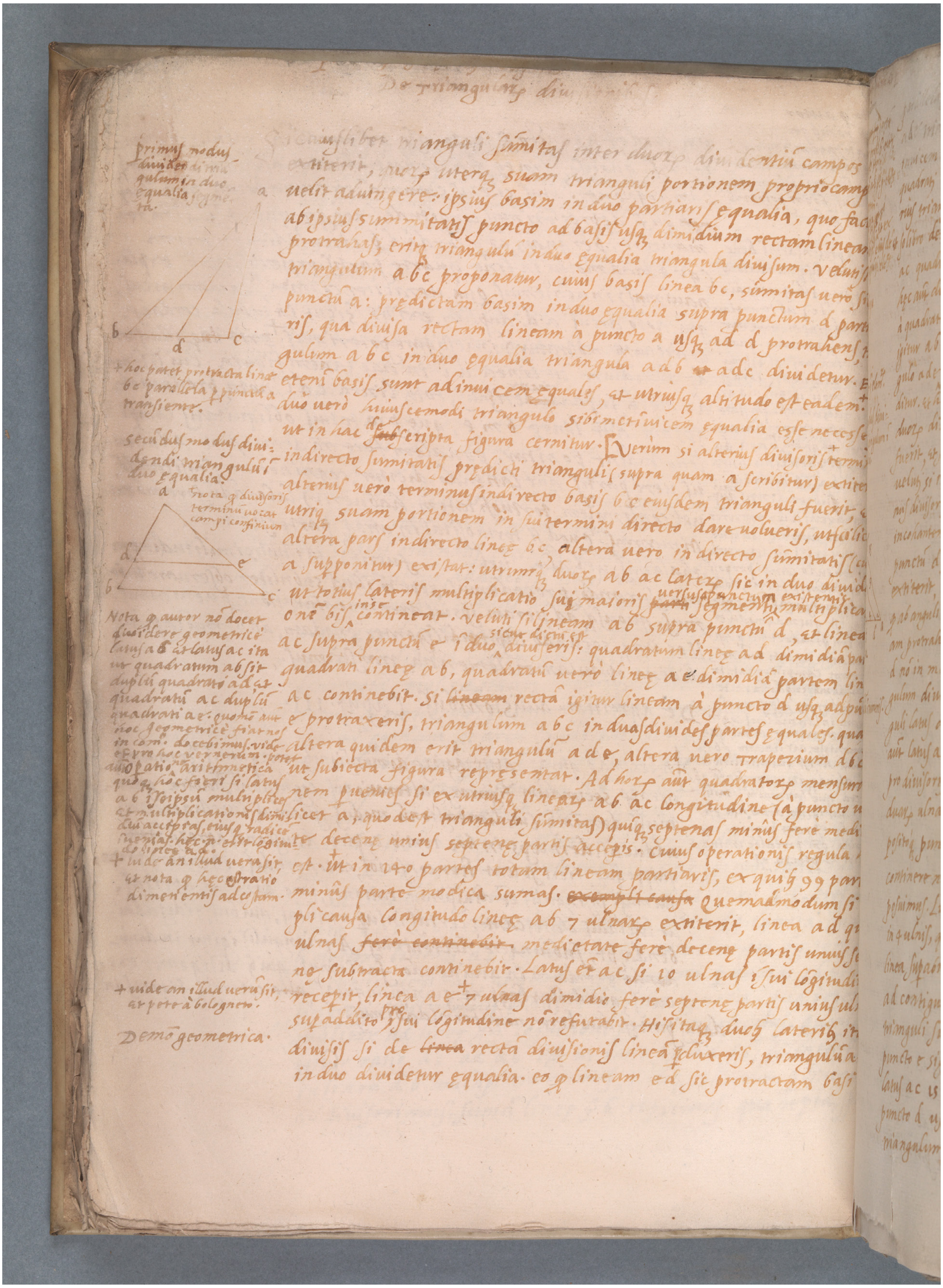

Fig. 1: Fol. 33v of D390, showing the various additions of Barozzi.

dealing with the measuring of heights and depths. F36 is the oldest manuscript known to contain *Liber embadorum*. It ends abruptly at the beginning of Book III, as will be elaborated below. It appears that a continuation once existed; however, the folios were torn from the manuscript. Manuscripts P7224 and P11246 are the ones used by Curtze, and both contain diagrams. They are the most complete among the Latin manuscripts, in the sense that there are (almost) no missing sections when compared to the Hebrew text. While P11246 contains no comments by the scribe in the margins, P7224 does contain several. These are either additions by the scribe after realizing that he forgot to copy (parts of) sentence(s), or comments concerning the text itself.

Having examined the general structure of the *Ḥibbur* and of *Liber embadorum*, the following section will focus on Book III of these works, which addresses the division of figures.

3. Book III of the *Ḥibbur* as a case study. Part 1: Towards a refined typology of textual transformations

The present and following sections constitute the core of this paper. This section examines the typology of textual changes across the various manuscripts, while the next section examines one specific manuscript, D390, which was copied and expanded by Barozzi through various comments and diagrams. As both sections take Book III of the *Ḥibbur* and *Liber embadorum* as a case study, which deals with the division of figures, I begin with a short overview of the historical background of this topic.

The Greek philosopher Proclus (410–485 CE), in his commentary on the first book of Euclid's *Elements*, mentions a book titled *On Divisions* (of Figures), which he attributes to Euclid.[40] The book itself deals with problems involving the division of a given figure into subfigures using one or more straight lines, where the resulting areas have various ratios to one another or to other given areas. While the original Greek text has been lost, an Arabic translation was produced during the tenth century by the Persian geometer Aḥmad ibn Muḥammad ibn ʿAbd al-Jalīl a-Sijzī. This contains statements of thirty-six propositions but only four proofs. The Arabic translation was discovered by Franz Woepcke, who translated it into French and published it in 1851. In 1915, Raymond Clare Archibald presented a 'restoration' of *On Divisions* based on Woepcke's translation and other works. Later, in 1926, Carl Schoy published a German translation of an anonymous tenth-century Arabic text titled *Various Geometrical Problems*, which also contains propositions from *On Divisions*.[41]

The Arabic works on this topic must have been known during the eleventh and the twelfth centuries, as parts of them, as well as comments on this topic and additional problems of it, are found in Book III of the *Ḥibbur*, and consequently also in *Liber embadorum*. Numerous elements of these Arabic works or of *Liber embadorum* are also found in Fibonacci's *De practica geometrie* (1220), whose chapter on the division of figures is more elaborated and contains additional material.

[40] See Moyon 2017, 43–44. On division of figures in the Arabic and Latin world during the Middle Ages, see Moyon 2017, 42–52 and 64–77.

[41] See Woepcke 1851; Archibald 1915; Schoy 1926. On the importance of this book, its sources and the historical background, see, among others, Hogendijk 1993; Moyon 2011.

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While the precise nature of the transmission of knowledge from *Liber embadorum* to *De practica geometrie* remains an open question, Barnabas Hughes identifies the propositions common to both treatises, thereby underscoring the possibility of such a transfer.[42] In this regard, the importance of examining the various Hebrew and Latin manuscripts of the *Ḥibbur* and *Liber embadorum* is evident: they are one of the first, if not the first, evidence of a translation of Euclid's *On Divisions* from Arabic to Hebrew.

In most manuscripts, Book III of the *Ḥibbur* is divided into three parts. The first part addresses the division of triangles, the second the divisions of quadrilaterals (into two, three, or four parts), and the last part focuses on the division of parts of a circle. It is important to note that manuscripts P995 and P1050 differ in both structure and content: the order of the sections is different; there is no separation into Books, and they contain only sections [129] to [141] of Book III (not including section [135], which serves as the introduction to the second part). These sections correspond to the first part of Book III and the initial portion of the second part, which addresses the division of a quadrilateral into two parts.

In this section, I examine Book III as a case study to introduce a more refined typology of textual transformations across the various manuscripts of the *Ḥibbur* and the *Liber embadorum*. In particular, I examine cases of textual reorganization, omissions and additions (mainly in form of macro-changes), as well as terminological and diagrammatic changes. The examination of the additions and omissions leads to a more nuanced typology, according to which such additions and omissions either result in a more theoretical point of view with respect to how the text should present mathematical knowledge, or, on the contrary, to a more practical (or pseudo-practical) orientation. In this context, the term 'practical' does not imply that the manuscript was necessarily intended for use by surveyors, constructors, or artisans. Rather, the exercises are presented in a less theoretical mode, omitting proofs and putting less emphasis on rigor. These types of omissions also reflect differing views on the degrees of solvability of mathematical problems. To illustrate this typology, I will examine the introduction to Book III, as well as sections [129] and [130] of the *Ḥibbur*, which correspond to sections 1–3 in Book III of *Liber embadorum*.[43] Sections [129] and [130] correspond to the first and second geometrical propositions in Book III. This close reading, focusing only on three sections, is essential for identifying the various changes and exploring the types of textual fluidity, as I will elaborate later. Section 3.1 analyzes the content of the introduction to Book III, as well as the first and second geometrical propositions on the division of triangles. Sections 3.2 and 3.3 examine the macro- and micro-changes found in the texts in the various manuscripts.

42 Hughes 2008, 185.

43 These are to be found in the following folios in the various manuscripts. In the Hebrew manuscripts: M256: fols 97ᵛ–98ᵛ; M299: fols 83ᵛ–84ᵛ; Pr2635: fols 49ʳ–50ʳ; P1048: fols 40ᵛ–41ᵛ; P1050: fols 39ʳ⁻ᵛ; P995: fols 271ʳ⁻ᵛ; P1061: fols 73ʳ–74ᵛ; V: fols 48ᵛ–49ʳ. In the Latin manuscripts: D390: fols 33ʳ–34ʳ; F184: fols 146ᵛ–147ᵛ; F36: fol. 35ᵛ (the copy ends at the middle of the third section); P7224: fols 43ʳ–44ʳ; P11246: fol. 26ʳ⁻ᵛ.

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3.1 The introduction and the first two propositions of Book III

Before presenting the typology of textual transformations among the various manuscripts, it is necessary to introduce the content of the introduction to Book III, as well as the first and the second geometrical propositions of this Book.

The introduction to Book III refers explicitly to Book II, noting that Book III addresses the divisions of lands and fields with various geometric shapes (e.g. triangles or quadrilaterals, discussed in Book II) among partners or heirs. It also mentions the proofs provided for the various propositions, so that readers may rely on and have confidence in them. The final phrase of the introduction indicates that Book III begins with an examination of triangular-shaped lands, then proceeds to quadrilaterals and other shapes.

This first geometrical proposition of Book III (section [129] according to Guttmann's and section 2 according to Curtze's system) poses the following task: to divide a triangle into two equal parts by drawing a line from one of the vertices. This proposition is accompanied by a diagram in nearly all the manuscripts, and almost all the manuscripts contain a drawing of what appears to be an isosceles triangle (see below). The procedure for solving the task is fairly simple. Given a triangle ABC, the manuscript copies of both the *Ḥibbur* and the *Liber embadorum* instruct the reader to draw a line from the vertex A to the midpoint D of edge BC. This results in two triangles, ABD and ACD, which have equal areas since they share the same height (from A) and their bases (BD and CD) are equal.[44]

The second geometrical proposition of Book III (section [130] according to Guttmann's and section 3 according to Curtze's system) poses the following task: to divide a given triangle ABC into two equal parts by drawing a line parallel to one of its edges. The solution is provided immediately after the problem is stated: choose a point D on AB and a point E on AC such that $(AD)^2 = ½(AB)^2$ and $(AE)^2 = ½(AC)^2$. The line DE partitions the triangle into two parts – triangle ADE and quadrilateral DECB – which have equal areas (see Fig. 2.1).[45]

Before the text proceeds to prove that DE indeed divides the triangle in the desired way, a numerical example is provided to illustrate how to mark, for example, the point D such that $(AD)^2 = ½(AB)^2$. Since AD = AB/√2 and AE = AC/√2, it is necessary to calculate the value of 1/√2. It should be noted that the notation √2 is, of course, modern and is employed here solely for simplicity. The copies of the *Ḥibbur* and *Liber embadorum* do not use such notation; instead, they describe the procedure in words. For instance, in the *Ḥibbur*, AbH instructs the reader to select a point D such that 'the square of AD is half of the square of AB'.[46] AbH asserts that it is known that 1/√2 is approximated by 5/7-1/140 = 99/140.[47] Hence, the following example is given: if AB has a length

[44] This problem appears also in Fibonacci's *De practica geometrie* and in Arabic works on the division of figures. See Hughes 2008, 186; Archibald 1915, 33, proposition 3; Hogendijk 1993, 148, proposition 1.

[45] This problem appears also in Fibonacci's *De practica geometrie* and in Arabic manuscripts on the division of figures. See Hughes 2008, 198–199; Archibald 1915, 30; Hogendijk 1993, 148, proposition 3.

[46] HMT, 83.

[47] While AbH does not specify explicitly here how he reaches this approximation, he states at the introduction to the *Ḥibbur* (HMT, 3) that he approximates √2 with the fraction 99/70. Since 1/2×√2 = 1/√2, one obtains that 1/√2 is approxi-

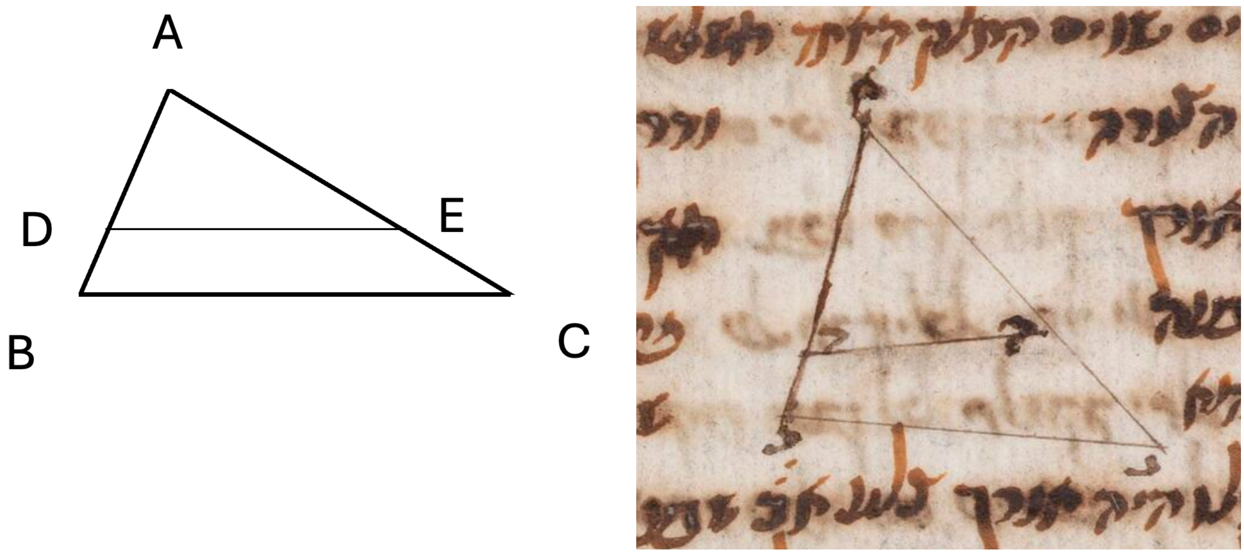

Fig. 2: (1) Diagram of a divided triangle into two parts with equal areas, as instructed in the second proposition of Book III (depicted by Michael Friedman); (2) the drawing on fol. 49v in Pr2635, where the line DE (in Hebrew letters דה) is not drawn as a parallel to BC.

of 7, then AD would be approximately 5-1/20, and if AC has a length of 10, then AE would have a length of (approximately) 7+1/14.

After presenting this numerical example to show how to (approximately) mark points D and E, the text turns to the proof. The claim, to recall, is that if D and E are marked as described above, then triangle ABC will be divided into two equal parts. At this point, AbH asserts that DE is parallel to BC, thereby inferring that triangles ADE and ABC are similar.[48] Taking this similarity into account, AbH explicitly refers to a proposition from Euclid's *Elements* (Book VI, proposition 19), mentioning Euclid by name:[49] the ratio of the areas of similar triangles is equal to the square of the ratio of their corresponding sides. Since $(AD)^2 = ½(AB)^2$ – or, equivalently, $2(AD)^2 = (AB)^2$ – the square of the ratio of the sides is 2, and it follows that the area of triangle ABC is twice that of triangle ADE. Consequently, the area of triangle ADE is equal to that of quadrilateral DECB.

3.2 Macro-changes

I now turn to the major textual changes observed when comparing the manuscripts, in order to demonstrate how these changes reflect the distinct character of each manuscript's content.

mated by 1/2×99/70 = 99/140 (see also section [46] of the *Ḥibbur*, in which AbH approximates √200 = 10√2 as 99/7). As we will see below, some Hebrew manuscript copies state that 99/140 is an approximation.

48 AbH's argument is not complete and logically misleading. In fact, the triangles ADE and ABC are similar because of the SAS similarity criterion (with the sides AD, AB; AE, AC and the common angle A). From this, it follows that ∢AED = ∢ACB, ∢ADE = ∢ABC and only therefore the line DE is parallel to BC.

49 Only Euclid's name and the content of the proposition itself are stated. Note that Euclid's name is also cited in section [106] in the *Ḥibbur* (which is in Book II). However, not all the manuscript copies mention Euclid in Book III (cf. n. 51). In *Liber embadorum*, the name of Euclid is already mentioned in Book I, at the end of section [25] (Curtze, 18).

3.2.1 Omission and reorganization of sections: More practical (Hebrew) manuscripts

Examining the Hebrew manuscripts, one realizes that the most significant changes appear in manuscripts P995 and P1050. Notably, these manuscripts omit the introduction to Book III. This is not surprising, given that, as noted above, the texts of P995 and P1050 exhibit a complete reorganization of Books II, III and IV compared to the other manuscripts, discarding the separation into books altogether in the process. Since P995 and P1050 can be considered as more practical manuals, in the sense discussed above, it is plausible that the introduction to Book III, which is more theoretical in nature, was considered unnecessary. Furthermore, as the title of Book III is missing, including the introduction would have been incongruous. Moreover, while the sections in Book III that immediately follow the introduction (= [129] till [134]) are indeed transmitted as a continuous unit, in both P995 and P1050 they are located in different places within the overall textual structure in between sections from Book II.[50]

Turning to the second proposition, manuscripts P995 and P1050 omit the proof of the statement and instead conclude the section with the numerical example.[51] This prompts the question: why was the proof not copied? One possible explanation is that the scribes had access only to a partial copy of the *Ḥibbur*. The scribe of manuscript P995 (but not the one of P1050) notes the following (fol. 269^{v}): 'Here I will write all of this from [the treatise on] geometry [*Tishboret*] and their shapes and forms / according to what I found letter by letter [faithfully] / and the first [Book] is missing'.[52] While the scribe acknowledges the absence of the first Book, implying that he was aware of its existence, it is possible that the other sections, too, were incomplete in the manuscript from which he copied. Secondly, most of the statements in P995 and P1050 are presented without proofs.[53] The presumed practical nature of the manuscripts' content may explain this: the section ends with the numerical example, this being the most immediately applicable part in the section. It is important to note that while the numerical example appears in all the other manuscripts (both Hebrew and Latin), its function there is primarily illustrative, as the proof supplies the theoretical foundation. Eleonora Sammarchi, in her analysis of the arithmetical-algebraic investigations of Abū Bakr Al-Karağī's (eleventh century CE) which are based on Book II of Euclid's *Elements* and in which some propositions are stated with a numerical example but without a proof, observes that in these cases 'the example becomes the element that guarantees the validity of the proposition. It [the numerical example] thus acquires a dual function: illustrative and argumentative'.[54] In this sense, while the proof is omitted, a different degree of solvability is introduced here in the form of an example

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[50] See Friedman and Garber 2023, 146.

[51] Hence, the scribes of these manuscripts also do not mention Euclid's name (while the name 'Euclid' is mentioned in all the other Hebrew manuscripts).

[52] "הנה אכתוב כל זה מתשבורת וצורותיהם כתבניתם / לפי מה שמצאתי אות באות / והראשונה חסרה". The adjective הראשונה (the first) is feminine, and therefore the Hebrew term for 'book' does not fit here, as it is masculine. According to the sentence, the most likely reference is to *Tsura*, צורה , 'shape'; however, it is clear from the content that the scribe is referring to the missing first Book.

[53] Friedman and Garber 2023, 144–148.

[54] Sammarchi 2025, 22.

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which is *both* illustrative and argumentative. I will return below to the role that such numerical examples played in the various manuscripts.

3.2.2 A more theoretical (Latin) manuscript: Criticizing the proof

Turning now to the Latin manuscripts, the most elaborate addition to the second proposition is found in D390, at fol. 33v, below the accompanying diagram (see Fig. 3 and Fig. 1 for the entire folio).[55] In this section Barozzi, the scribe of the manuscript, offers a critique of the manner in which Plato of Tivoli (or AbH) presented the arithmetical example, that is, the practical solution. Barozzi's comment begins as follows: *Nota quod autor non docet dividere geometrice* ('Note that the author does not teach how to divide geometrically'). While the main text shows how to find the segments AD and AE *arithmetically* – by approximation, that is, by multiplying the lengths of AB and AC with 99/140 – Barozzi observes that no geometrical method is provided for constructing the segment AD (or AE) from AB (or AC, respectively). In this way, Barozzi criticizes the solution proposed by AbH (and, by extension, by Plato of Tivoli), emphasizing that a mathematical problem may be solved in different ways, each involving a different *degree* of solvability: it may be practically solvable (by means of arithmetical calculations only, as in manuscripts P995 and P1050), or it may also be geometrically solvable, and thus more theoretically rigorous.[56]

3.2.3 Diagrammatical changes and the addition of comments to diagrams

Before examining the changes in the diagrams, let us first see which manuscripts actually contain them. With regard to the first proposition in the manuscript copies of the *Ḥibbur*, only manuscript M256 does not contain a diagram. However, a blank square on fol. 98r indicates that a diagram – presumably that of the divided triangle – was intended to be drawn. All the other manuscripts contain a drawing of what looks like an isosceles triangle (see for example Fig. 4).

In the Latin manuscripts, the diagram accompanying the problem appears in manuscripts F36, P7224, P11246, and D390. F184 contains no diagrams at all. However, the end of the section does

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[55] Moreover, though outside the scope of this paper, one should note that the manuscript copies of *Liber embadorum* contain a section in Book III (after the first two propositions) that does not appear in the *Ḥibbur*. Indeed, section [133] of the *Ḥibbur*, which is equivalent to section [6] of Book III of *Liber embadorum*, deals with the task of dividing a triangle into three parts of equal area, when the first part contains an edge of the triangle, and the other two parts the opposing vertex. Section [133a] of the *Ḥibbur* presents a numerical example, which is presented in the continuation of section [6] of *Liber embadorum*. However, this numerical example in *Liber embadorum* is more elaborated, containing explanations which do not appear in any of the Hebrew manuscripts. Moreover, section [7] of Book III in *Liber embadorum* does not appear at all in the *Ḥibbur*: it contains an additional way to solve the problem of dividing a triangle into three parts of equal area.

[56] It is moreover important to underline that while such more theoretically rigorous method is not explicitly explained in this short note, it was well known at that time and presented in the *Ḥibbur* and in *Liber embadorum*. Recalling that what is asked for is to construct a segment AD such that, given AB, $(AD)^2 = \frac{1}{2}(AB)^2$, such a task is equivalent to the following: given a square of area S, construct another square of area S⁄2. This problem is already presented and solved geometrically with the Pythagorean theorem; to explicate, given an isosceles right triangle whose legs are of length X (and hence the area of a square built on it is X^2), the area of the square built on it is $2X^2$. This problem is presented, both as an arithmetical and a geometrical one, as one of the first problems in Book II (in *Liber embadorum*, see Curtze, Book II, section 8; in the *Ḥibbur*, see HMT, section [46]).

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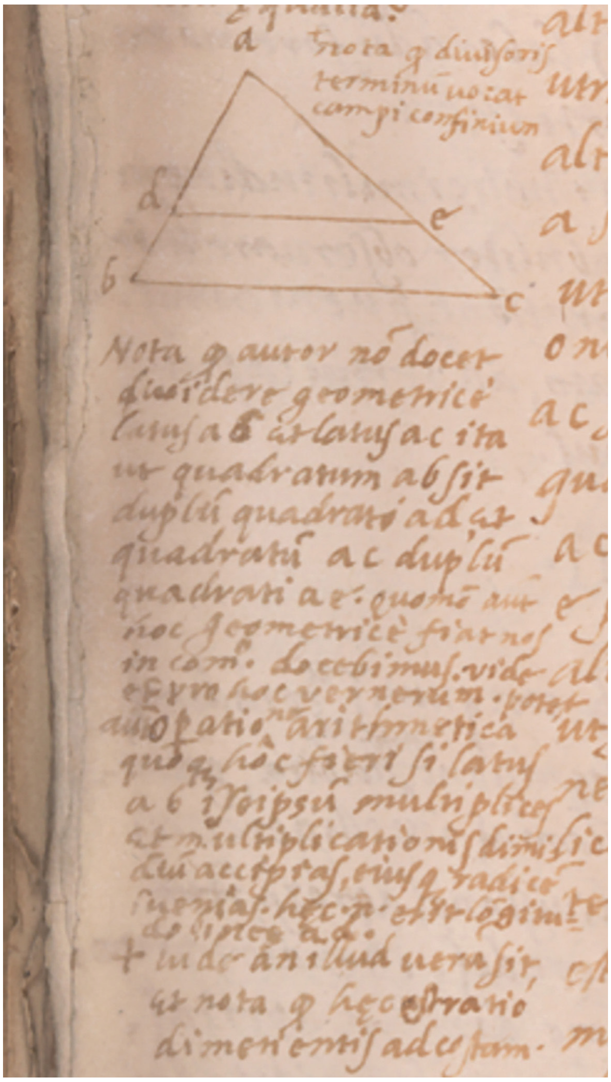

Fig. 3: The lower part of the left margins of D390, fol. 33v; Barozzi's note below the diagram critiques the method presented to solve the proposition.

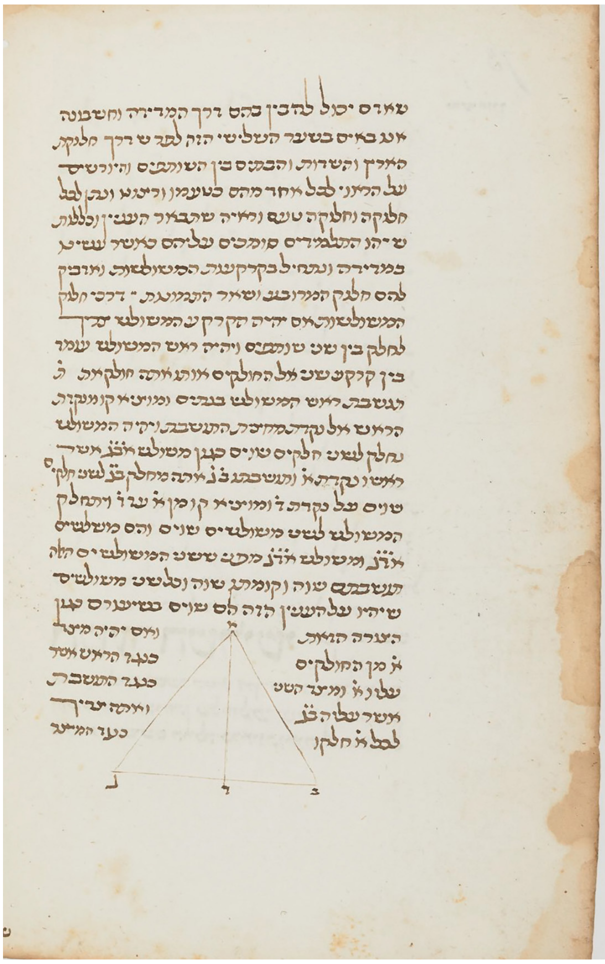

Fig. 4: The drawing of a divided triangle ABC (in Hebrew letters: אבג) in the manuscript P1061, fol. 73v.

refer to a 'figure' (that is, a diagram) and this reference was copied by the scribe of F184. This may suggest that the scribe was at least aware of the existence of the diagrams. In manuscripts F36, P7224 and P11246, the diagram that accompanies the text also depicts an isosceles triangle. Barozzi, who likely noticed that an isosceles triangle might be misleading for an inexperienced reader, as I will explain below, draws an obtuse triangle in D390 (see Fig. 5 and Fig. 1 for the entire folio). Ken Saito and Nathan Sidoli, in their work on diagrams in the Euclidean tradition, observe that using over-symmetrized diagrams was a common practice.[57] Despite the absence of explicit

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[57] See Saito and Sidoli 2012, 157: 'The two most prevalent characteristics of the manuscript diagrams are […] overspecification [here in terms of symmetry (or over-symmetry)] and indifference to [metrical, geometrical or] visual accuracy. The consistent use of overspecification implies that the diagram was not meant to convey an idea of the level of generality discussed in the text. The diagram simply depicts some representative examples of the objects under discussion and the fact that this example is more regular than is required was apparently not considered to be a problem'. See also Netz 1998.

statements from the scribes regarding the use of such symmetrical diagrams, one may assume that Barozzi intended to signal that over-symmetry could be misleading.

Moreover, Barozzi adds two marginal notes along and below the diagram. The first is a subtitle on 'the first method for dividing a triangle into two equal parts' (*primus modus dividendi triangulum in duo equalia segmenta*; see Fig. 3), which pertains specifically to this section. The second is a comment on the statement that 'the height of both [triangles ADB and ADC] is the same' (*et utriusque altitudo est eadem*). Barozzi inserts the symbol + at the end of the sentence, and writes the following note at the margins : + *hoc patet protracta linea bc parallela per punctum a transiente*. This is a direct reference to the diagram (see Fig. 5), specifically the diagram of the obtuse triangle. Barozzi's note indicates that the height, exiting from (*transiente*; literally, passing through) the vertex A, is perpendicular to the continuation (*protracta*) of the edge BC.

What do these additions indicate? Barozzi's decision to draw an obtuse triangle suggests that he understood the need to demonstrate to the intended readers of the text how the division might appear in triangles that are not isosceles. These readers, who Barozzi expected to read the printed version of the *Liber embadorum*, may have included mathematicians, but more likely consisted of students, since the exercises are relatively elementary. Had these readers lacked any mathematical education, they might still have understood intuitively why, in the case of an isosceles triangle, the two resulting triangles would have the same height (as it is clearly illustrated). In an obtuse triangle, however, this claim is not so easily visualized, as the height lies outside the triangle.

Turning now to the second proposition, and beginning with the Hebrew manuscripts of the *Ḥibbur*, it is noteworthy that only M256 does not contain a diagram, but, once again, features an empty square in which a diagram should evidently have been drawn. In Pr2635, the line DE is not drawn as a parallel line (Fig. 2.2), which likely reflects an unintentional error in its rendering or, less probably, a deliberate choice by the scribe to highlight that, from the initial construction, it is not clear that such a line would be parallel.

Examining the Latin manuscripts, one notes that F36 and F184 do not contain diagrams. Furthermore, while D390 includes a single diagram (as is the case in the Hebrew manuscripts), P7224 and P11246 each contain two diagrams (Figs 6.1 and 6.2), though only one is correct. It is important to note that Curtze, who based his printed edition of the *Liber embadorum* on P7224 and P11246, only included the correct diagram without any mention of the additional, incorrect one that was drawn by the scribes of those manuscripts.[58] Why were these two diagrams drawn? Fig. 6.2 displays the correct diagram, while Fig. 6.1 shows a 'misleading' diagram in which points D and E are depicted as the midpoints of AB and AC, respectively. One may surmise that the scribe responsible for the version transmitted in P7224 and P11246, while copying the statement and perhaps lacking mathematical training or failing to continue reading the full explanation, misinterpreted the phrase 'the square of AD is half of the square of AB'. If the scribe overlooked the term 'square', he may have misread the statement as 'AD is half of AB'. However, this could also reflect the broader tradition of over-symmetrizing Euclidean diagrams, as discussed above.

[58] Curtze, 130.

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Fig. 5: A detail of D390, fol. 33ᵛ with the diagram and the marginal comments of the scribe concerning the first proposition. The uppermost comment asserts that this is the first way to divide a rectangle (into two equal parts). The comment below the diagram refers to the height from A, when in this specific case, it meets the *continuation* of the edge BC.

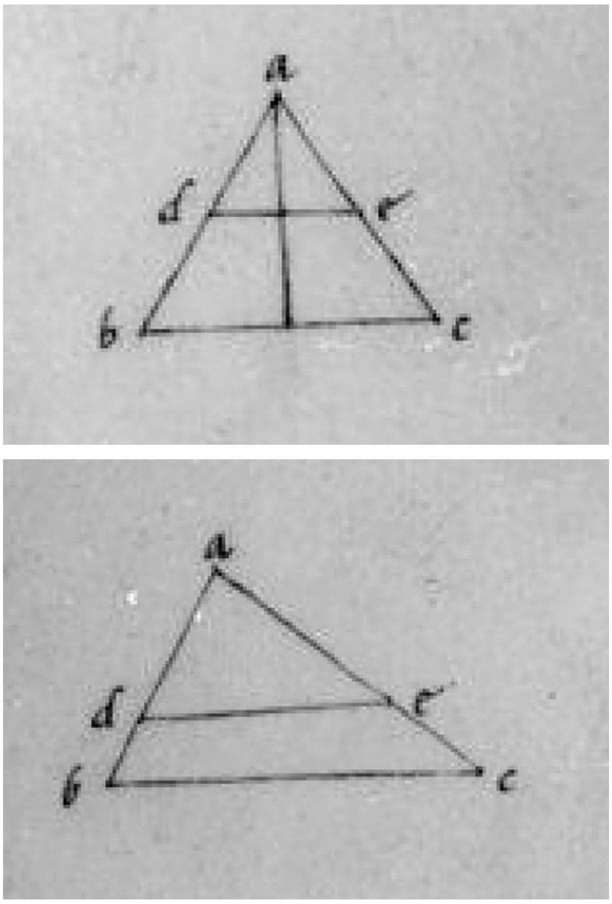

Fig. 6: The two diagrams presented in P7224 for the second proposition (above [1]: fol. 43ᵛ, below [2]: fol. 44ʳ). Similar diagrams are found in P11246, fol. 26ᵛ.

* * *

In summary, two main tendencies can be identified in how the contents of the *Ḥibbur* and *Liber embadorum* were modified. First, some manuscripts present a more practical or rather, a less theoretical version of the text. This is the case in the Hebrew manuscripts P995 and P1050, which omit the proofs and the introduction in Book III, leaving the reader with numerical examples that both illustrate and assert the validity of the propositions. Conversely, the texts in other manuscripts take a more theoretical-critical approach. This is exemplified by the Latin manuscript D390, in which the scribe adds a critical assessment of the diagram or the method of proof. These two tendencies reflect various dimensions or degrees of solvability: either through numerical examples but without accompanying proof, or through theoretical proofs, some of which even call into question the adequacy of the original demonstrations. Further changes, observed in several other manuscripts, include the over-symmetry of diagrams, or, in one case, criticism on it, and were also commonplace in the tradition of drawing Euclidean diagrams.

3.3 Micro-changes

Considering first the macro-changes at the content level, we have seen various procedures employed to render a manuscript's content more practical or more scholarly. These include reorganization, omissions of proofs, modifications of diagrams, and critical engagement with existing proofs. The following analysis turns to micro-changes, which offer, at least in part, evidence of similar transformations aimed at enhancing either the practical applicability or the scholarly rigor of the text. As these changes are of more subtle nature, I will present them in the order in which they appear in the *Ḥibbur* and the *Liber embadorum*, respectively: beginning with the introduction, followed by the first and second propositions.

(i) Concerning the introduction, it is worth noting that the Hebrew manuscripts Pr2635 and M256 include the following justification for the inclusion of proofs: 'שלא יתאונה אחד מהם בחלוקותם' ('In order that one will not complain on [the validity of] their partition'). This expression does not appear in the other Hebrew manuscripts, nor is there any equivalent found in any of the copies of the *Liber embadorum*. The emphasis on the validity of the arguments recurs later in these two manuscripts, which also emphasize the more scholarly and general character of the treatise in other sections. This particular addition emphasizes not only the rationale for including the proofs but also positions the texts presented in these two manuscripts as being more theory-oriented. As previously noted, Pr2635 and M256 contain proofs for all propositions in sections [30]–[41] of Book I, while the other Hebrew manuscripts do not.[59]

With regard to textual differences in the introductions of the Latin manuscripts, one notes that while the last phrase of the *Ḥibbur*'s introduction to Book III announces that the book begins with propositions concerning triangular-shaped lands, followed by quadrilaterals and other shapes, the introduction of the *Liber embadorum* additionally specifies that Book III will also address pentagon-shaped fields. Most manuscripts use the term *pentagonas*; the sole exception is D390, in which Barozzi uses *quinquangulas*, a Latinization of the former.[60]

(ii) The first proposition concerns the division of a triangle in two equal parts by drawing a line from one of its vertices. In the Hebrew manuscripts M299 and V, we find a phrase justifying the choice of the vertex A when performing this division: 'כדי שירצו שיהיה חלק כל אחד מהם עם קרקעו האחר בצד אחד' ('So that they want each of their parts to be on a different side').[61] This phrase does not appear in the other Hebrew manuscripts. Notably, an equivalent sentence is contained in all the manuscript copies of the *Liber embadorum* at the corresponding section, which may suggest that M299 and V are the closest to the Hebrew version from which the Latin translation was derived.

Additionally, at the end of this section, immediately following the proof, Pr2635 and M256 include the following phrase: 'כאשר מסרנו בכללות בראש הספר' ('As we have informed [noted] generally at the beginning of the book').[62] This statement suggests that the person responsible for this version

[59] See also Friedman and Garber 2023, 138–140.

[60] Such Latinizations are seen also in other terms employed by Barozzi.

[61] Cited from V, fol. 48v.

[62] Cited from Pr2635, fol. 49v.

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of the *Ḥibbur* was aware that even this simple, practical exercise carries a general justification. Once again, this phrase emphasizes the more theoretical character of the texts in Pr2635 and M256.

In the Latin tradition, in D390, Barozzi assigns a heading to the entire set of propositions concerning the division of triangles: *De triangulorum divisoribus* ('On the division of triangles'). This title appears at the top of folio 33^{v}, while in the other manuscripts it is found in the text itself (and not as a title) just after the proposition is stated.

(iii) Another notable textual change occurs in the second proposition, which concerns the construction of a segment in a given triangle that is parallel to one of the edges and divides the triangle into two equal areas. One of the steps instructs the construction of a section AD such that $AD = 1/\sqrt{2}\,AB$, using the approximation $1/\sqrt{2} \sim 99/140$. Among the Hebrew manuscripts, only M299 and V explicitly note that $1/\sqrt{2}$ is less than 99/140. For example, in manuscript Pr2635 (as well as in the other manuscripts) one finds the following statement: 'והכלל המסור לזה הוא שתהיה לוקח מהקו צ"ט חלקים מק"מ חלקים בו' ('And the rule given to this [for calculating $1/\sqrt{2}$] is to take from the line 99 parts out of its 140 parts').[63] In M299 and V, however, the expression is more nuanced: 'והכלל המסור לזה שתהיה לוקח מן הקו צ"ט פחות משהו מן ק"מ חלקים בו' ('And the rule given for this is to take from the line *less* than 99 parts from its 140 parts').[64] In other words, it is necessary to subtract a small amount from 99/140 to obtain $1/\sqrt{2}$, though it is not specified what the exact value of the subtraction is. The absence of this clarification in the other manuscripts is striking, given that anyone with a reasonable mathematical education would recognize that an irrational number cannot be equal to a rational number, but only approximated.[65] This omission becomes even more noticeable when considering section [46] in the *Ḥibbur*, in which AbH approximates $\sqrt{200} = 10\sqrt{2}$ with 99/7, and explicitly notes that $\sqrt{200}$ is less than 99/7. This comment appears in *all* the Hebrew manuscripts. Therefore, since all the manuscript copies address the issue of the numerical approximation of geometrical magnitudes in this section, it is plausible that the omission concerning the calculation of $1/\sqrt{2}$ originated from a single manuscript, from which all the known Hebrew manuscripts except M299 and V originated.

Finally, it is essential to highlight a few differences that arise when comparing the copies of the *Liber embadorum* to the Hebrew manuscripts. First, all Latin copies explicitly state that one must subtract a small amount (*minus parte modica*)[66] from 99/140 to obtain $1/\sqrt{2}$. This detail strengthens the hypothesis that the Latin translation was based on the version of the *Ḥibbur* from which M299 and V originated. Second, when introducing the proof, the Latin manuscripts note that the line DE is parallel to BC due to 'geometrical reasons' (*geometricalis*).[67] This particular expression does not appear in the Hebrew manuscripts, which suggests that Plato of Tivoli intended to emphasize that the reasoning presented in this section relies on geometric principles not explicitly laid out in

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63 Cited from Pr2635, fol. 49^{v}.

64 Cited from V, fol. 48^{v}. Italics by the author.

65 Two options may be possible assuming that an *Ur-version* (i.e. only one version) existed: first, the version, from which M299 and V originated, completed this expression (that is, in the *Ur-version* the expression was incomplete); second, the other manuscripts omitted this expression (hence in the *Ur-version* the expression was complete).

66 Curtze, 132.

67 Curtze, 132.

the proof. Interestingly, as previously noted, Barozzi added a remark on the solution to the second proposition, noting that 'the author does not teach how to divide geometrically'.

* * *

To summarize, the micro-changes both support the findings of our analysis of the macro-changes and reveal additional aspects of the manuscripts' content. First, we have seen that some of the manuscripts with versions of the text that exhibit a more theoretical character, Pr2635 and M256, as well as some of the Latin manuscripts, contain remarks that point out the approximate nature of the numerical example or emphasize various 'geometrical reasons'. However, while all the manuscripts contain the numerical example in the second proposition, not all of them explicitly mention its approximate nature. This omission is perhaps unsurprising, since the approximated nature was already noted before, but whether it resulted from deliberate choice or a transmission error, it once again illustrates the various degrees of solvability presented across the extant manuscripts. Second, we have identified one manuscript, D390, as having a more readership-oriented conception of the content, as evidenced by the addition of titles and the Latinization of terms. This reinforces the view that the scribe, Barozzi, may have intended this copy to be the basis for a printed edition of the *Liber embadorum*. Moreover, we have highlighted textual features that support the hypothesis that manuscripts M299 and V are the closest to the Hebrew source from which the Latin version was translated.

That said, it is important to note that the changes discussed here, as well as in the previous section, should not be interpreted as clear indications of scribal intention as though all the scribes were 'active modifiers' of the content. Some of them were, however. The scribe of P995, for instance, explicitly notes that Book I is missing (see above, Section 3.1), thus acknowledging a departure from the original structure. In D390, Barozzi stands out as a clearly identifiable 'critical editor', whose interventions go far beyond mere transmission. His editorial work in D390 will be discussed in the following section.

4. Book III of the *Ḥibbur* as a case study. Part 2: Zooming in on Barozzi's additions to Book III of *Liber embadorum*

Having examined three sections from Book III, I would like to take a step back and turn to Barozzi's additions in D390, situating it within the context of its production. It should be noted that since D390 was copied in 1565, one can assume that Barozzi was aware of Fibonacci's *De practica geometrie* (and potentially of other books and texts dealing with the division of figures).

Barozzi produced his manuscript as part of his broader project to make mathematical texts written in other languages more easily accessible to Latin readers.[68] His notes on the text of the *Liber embadorum* reflect the state of the art of practical geometry and the interest in the division of

[68] On Barozzi's translation of Proclus' commentary, mentioned above, see Rizzi 2017, 45–48. Of course, Barozzi was not the only one who promoted the dissemination of ancient and medieval mathematical knowledge; see, for instance, Henry Billingsley's first translation into English of Euclid's *Elements*, published in 1570 (Billingsley 1570).

figures in the fifteenth and sixteenth centuries, as evidenced by the publication of Commandino and John Dee of *De superficierum divisionibus* attributed to Machometus Bagdedinus (Muḥammad al-Baghdādī; 1050–1141), in 1570.[69] Clavius emphasized the role of practical mathematics in the mathematical curriculum and authored various practical mathematical treatises. Among these was the 1604 *Geometria practica*, which considers the division of polygons and contains numerous references to and comments on Euclid's *Elements*.[70] While a comparison of these treatises with the comments in D390 in Book III is beyond the scope of this paper (even though it remains a much-needed undertaking) they highlight the state of and interest in practical geometry in Europe in the mid-sixteenth century.

Returning to Barozzi's additions in Book III, which are consistent with his interventions throughout the manuscript, one notes that in addition to the added commentaries, titles, and subtitles discussed above, there are also 'numerated' diagrams: diagrams with numbers indicating the length of various segments that accompany numerical examples in the text (Fig. 7). The *Ḥibbur* itself contains numerical examples for the geometrical propositions. These examples were central not only for the propositions themselves but also to illustrate that the propositions may be useful for measuring and surveying. The fact that Barozzi emphasizes such numerical treatment, which is in keeping with practical geometry treatises during this period, indicates that he situated the text within this genre, thus reflecting a practice that 'occurred with increasing frequency throughout the early modern Euclidean tradition'.[71] This is also seen in the small note glued to fol. 35ʳ, which consists of arithmetical calculations pertaining to one of the sections of *Liber embadorum* (Fig. 7). However, it remains unclear whether these calculations were meant to be incorporated into the printed edition or whether these were Barozzi's own working notes, intended only to verify the exercises presented in the text.

It should be noted that the use of numbers and inclusion of numerical examples in geometrical exercises and propositions was common practice in the *Ḥibbur* and *Liber embadorum*, as was already mentioned above, and this practice characterizes medieval mathematical works both in theoretical and practical geometry.[72] Given that Barozzi also stresses in his comments the theoretical side of *Liber embadorum*, it seems that he challenges the separation between the theoretical and the practical aspects of mathematics that existed in earlier mathematical manuscripts. Barozzi also emphasizes on fol. 33ᵛ the need to provide a proper proof, which is associated to a theoretical approach to geometry, by underlining that the arithmetical construction lacks a geometrical explanation. By including this comment, he also underlines the various degrees of solvability of a mathematical problem.

Barozzi's emphasis on the theoretical aspects of geometry is further evidenced by his repeated references to Euclid's *Elements* in the paracontent of D390. In the margin of fol. 34ʳ, Barozzi

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69 Note that already in 1563, John Dee passed a Latin copy of *De superficierum divisionibus* to Commandino. On practical geometry during the sixteenth century, see Axworthy 2022. On Commandino and John Dee, see Rose 1972.

70 On Clavius and especially on his treatise on division of polygons, refer to Knobloch 1997; Knobloch 2015.

71 Axworthy 2022, 75. As Axworthy shows (2022, 75–83), various mathematicians – such as Billingsley, Clavius, Scheubel and Xylander – treated numerically geometrical propositions in treatises on practical geometry.

72 To quote only two recent works, among others, on the usage of arithmetic in Arabic, Latin and vernacular in the medieval traditions of Euclid's *Elements*, I refer to Corry 2013 and Sammarchi 2025.

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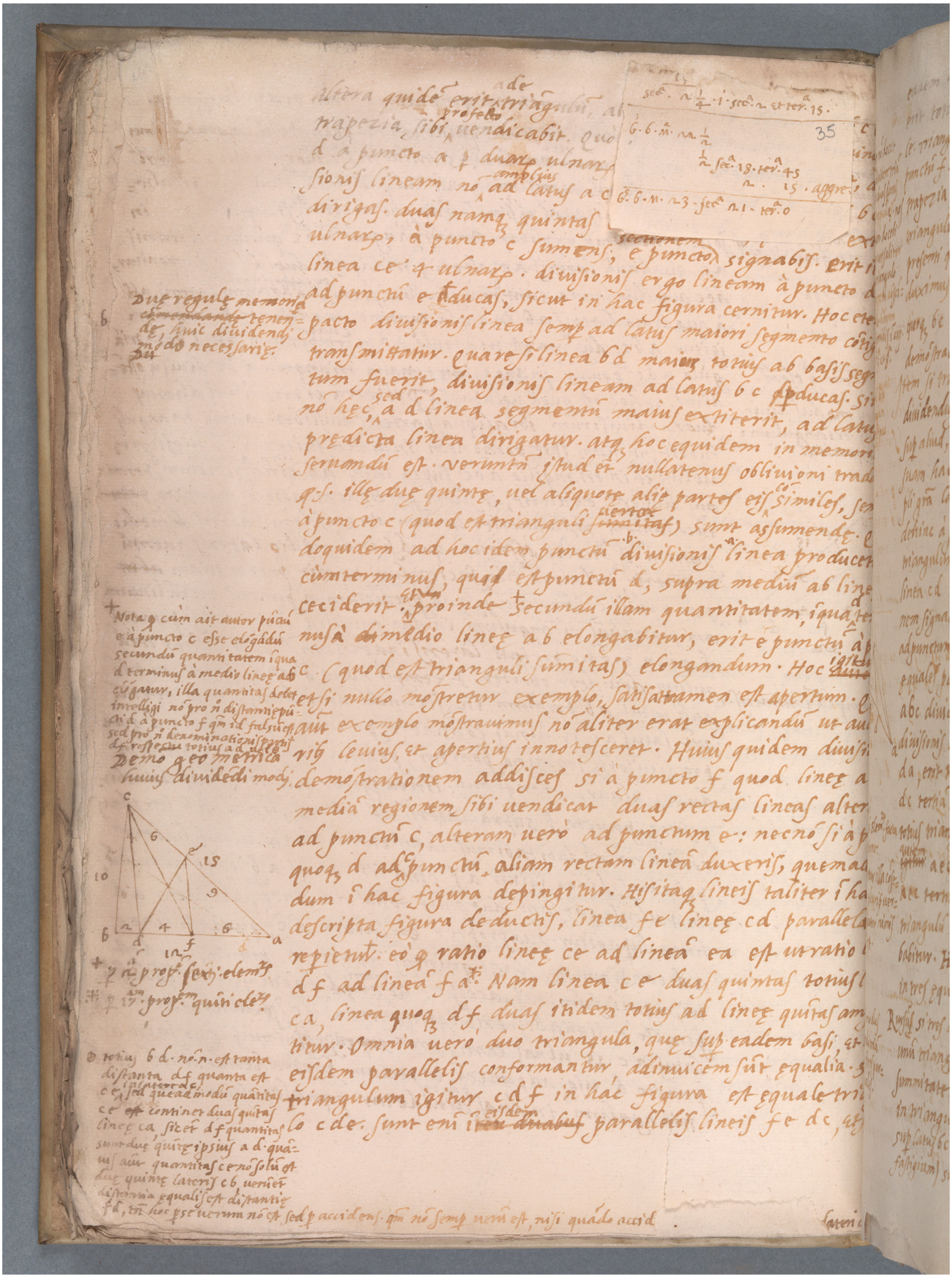

Fig. 7: Folio 35[r] of the manuscript D390. It shows two unique characteristics of Barozzi's additions: first, the numerated diagrams (on the left margins, below), and second, his glued note with numerical calculations (on the top of the folio).

has added a note to draw attention to the text's reference to Euclid in its proof of the second proposition. This is not the only instance in which he added references to Euclid's work. For example, on fol. 4r, which contains a portion of Book I, one finds Barozzi's notes next to several propositions, indicating their correspondence to propositions 35, 36, 37, 38, and 46 of Book I of Euclid's *Elements*. These explicit additions once more highlight Barozzi's effort to situate the *Liber embadorum* within a more theoretical framework, while also pointing towards the preparation of a printed edition. They show his attempt to situate *Liber embadorum* in a tradition which refers to Euclid's *Elements* by adding references which hardly existed in the Hebrew witnesses (or in the other Latin manuscripts). Equally important, Barozzi's references to Euclid help us identify the intended readership: although Barozzi's future printed edition was likely intended to contribute to the growing corpus of works on practical geometry, his emphasis on theoretical aspects and references to Euclid also suggest a desire to position the *Liber embadorum* as engaging with, or at least partially based on, the *Elements*. However, since these additions include practical, numerical, and theoretical aspects, there is no clear-cut separation between these categories. This aligns with the efforts of other Renaissance and early modern mathematicians, who sought to unite these traditions, or more specifically, to erode the distinction between theoretical and practical geometry.[73]

5. Conclusion: Types of changes of medieval mathematical manuscripts

What do these various changes reveal about how medieval mathematical manuscripts were copied and read, particularly in the case of the *Ḥibbur* and *Liber embadorum*? First and foremost, the close reading undertaken in this paper suggests with a high degree of probability that the Latin translation is most closely aligned to the Hebrew manuscripts M299 and V. That is, the Latin translation was probably prepared from a textual ancestor from which M299 and V derive. Given that the Latin translation is known to have been produced soon after AbH's text, one may further postulate that M299 or V are the closest extant witnesses to that original.

Second, focusing initially on the Hebrew manuscripts, it is evident that they exhibit a variety of textual modifications both on the macro- and micro-levels. On the micro level, one notes omissions of expressions which have mathematical relevance. This may indicate that some scribes, though not necessarily those who copied the manuscripts examined here, did not pay sufficient attention to the mathematical content they copied. Conversely, some additions emphasize the nature of the mathematical propositions, thereby highlighting a more theoretical orientation of some versions, in particular in Pr2635 and M256. These additions show a degree of scholarly engagement with how the content should be presented. Most of the scribes of the Hebrew manuscripts, as well as the Latin ones, however, faithfully transcribed the texts, thus continuing the transmission of earlier versions, even though various Hebrew manuscripts exhibit certain degrees of textual fluidity. For example, when considering the macro-changes evident in P995 and P1050, we encounter not merely a reorganization of the *Ḥibbur*, but a substantial abbreviation of it. These manuscripts stem from a version of the *Ḥibbur* in which the scribe(s) chose to redact and shorten the text,

73 Axworthy 2022, 88–92.

thereby rendering the mathematical content more practical or, more precisely, less theoretical, for the benefit of students who wished to, or were required to, practice problem-solving without engaging with the accompanying proofs. In this sense, one identifies here a more 'pseudo-practical' or 'mathematically practice-oriented' tendency. The absence of various sections from P995 and P1050 – the introductions to the individual books, Book I itself, and the proofs accompanying many propositions[74] – may suggest that both manuscripts were conceived as textbooks for learning practical geometry. The omitted sections typically contain introductory, apologetic, or theoretical content, which may have been considered irrelevant in a didactic context focused more narrowly on exercises and procedures.

Among the Latin manuscripts, D390 stands out. Unlike the Hebrew manuscripts, this is an annotated version prepared for printing, with comments and additions written in the margins. Barozzi's additions, which concern both the core content and paracontent, actively modify and comment on the text. The main body of the text is altered only to the extent of eliminating doublets and latinizing the Greek terminology. However, the paracontent added by Barozzi structures and critiques the mathematical content, explicitly highlighting varying degrees of solvability while simultaneously blurring the distinction between the practical and the theoretical dimensions of the mathematical content of *Liber embadorum*. Barozzi's additions reveal his extensive mathematical knowledge: his addition of the diagram of the obtuse triangle and his observation that the arithmetical construction lacks a geometrical explanation reflect a commitment to a classical standard that is absent in (pseudo-)practical traditions. However, Barozzi's additions do not exhibit the same 'fluid' nature as the significant textual changes observed in the Hebrew manuscripts, especially in P995 and P1050. Barozzi's additions reveal that he saw himself not merely as a scribe but as an editor. His goal was to produce an annotated critical text that balanced practical, numerical, and theoretical elements suitable for printing and wider dissemination. Accordingly, he tailored his edition to the mathematical standards of a Latin readership of mathematicians and students.

Changes such as Barozzi's critique on the solution of the second proposition in Book III, as well as the omissions of proofs in P995 and P1050 (or even the minor theoretical statements added in Pr2635 and M256), point towards differing degrees of solvability for (mathematical) problems and consequently, to different intended readers. What suffices for a text on practical geometry, e.g., simple arithmetical examples, may be inadequate in a context where theoretical rigor is emphasized. Ultimately, the various comments, additions, and omissions found in these manuscripts suggest that mathematical problems, and indeed mathematical texts, were not regarded as fixed or immutable entities. Rather, they were perceived as dynamic and adaptable, evolving from one manuscript to another. In this light, the examination of Book III also calls for an expanded typology of textual changes, as these shifts reflect not only different modes of reception and use of the texts but also broader developments within the tradition of practical geometry in early modern Europe.

[74] In fact, in these manuscripts also the preface and the (non-mathematical) conclusion are not copied.

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Abbreviations

HMT: Bar Hiyya, Abraham (1912–3), *Chibbur ha-Meschicha weha-Tischboreth : Lehrbuch der Geometrie des Abraham bar Chija*, ed. Michael Guttmann, Berlin: Vereins Mekize Nirdamim (in Hebrew).

Curtze: Curtze, Maximilian (1902), 'Der 'liber embadorum' des Savasorda in der Übersetzung des Plato von Tivoli', *Abhandlungen zur Geschichte der Mathematischen Wissenschaften*, 12: 1–183.

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