Educational Design Research

Volume 9 | Special Issue 1 | 2025 | Article 77

Contribution Academic Article

Title

Contributions from the use of photographic images in math classes: building knowledge through Educational Design Research

Author

Débora de Sales Fontoura da Silva Frantz
Universidade Franciscana – UFN
Brazil

Vanilde Bisognin
Universidade Franciscana – UFN
Brazil

Abstract

This study presents partial results of a research that aims to analyze the contributions that an educational product, built from photographic images, can provide to elementary school mathematics teachers, in continuing education, in the construction of geometry concepts. We used the methodological approach of Educational Design Research according to McKenney and Reeves (2019). This approach involves three phases: analysis and exploration of the problem; design, construction and application of an educational product; evaluation and reflection on the whole process. From a Math Trail carried out in the city's streets and cathedral, images were obtained that were used to design the activities that made up the educational product. Data was collected by means of video recordings, a questionnaire applied to the teachers taking part in the research and materials created by the teachers. From the evaluation and reflection, the following design principles were defined: a) the use of photographic images and the use of technological resources facilitate the understanding of geometric concepts; b) relating art and mathematics enhances visualization and facilitates the exploration of the geometric properties of flat figures; c) activities designed to take into account the local reality and its cultural aspects are motivating for the teaching of geometry.

Keywords Design principles
Mathematical trail
Photographic images
Visualization
Geometry teaching

DOI dx.doi.org/10.15460/eder.9.1.2155

Citation Frantz, D.S.F.S., Bisognin, V. (2025). Contributions from the use of photgraphic images in math classes: building knowledge through educational design research. *EDeR - Educational Design Research*, *9*(1), 1-27. http://dx.doi.org/10.15460/eder.9.1.2155

Contributions from the use of photographic images in math classes: building knowledge through educational design research

Débora de Sales Fontoura da Silva Frantz, Vanilde Bisognin

1.0 Introduction

The subject of the qualifications of teachers who teach mathematics and their difficulties is a recurring one and has been the subject of research in the literature at national and international level in recent times. These studies include the work of Lorenzato (2015) at a national level and, at an international level, the work of Rodrigues and Ponte (2020), among others. According to these authors, different paths are listed that can contribute to the qualification of teaching, among which are the updating of curricula and the development of projects that seek innovation, both from the point of view of content, methodologies or the organization of teaching-learning activities. According to Van den Akker (1999) and Van Driel et al. (2005), much of the existing research is dedicated to studying the role of teachers, new strategies for teaching specific content, the use of digital tools and the actual work carried out with students in the classroom. When dealing with strategies for teaching mathematics, it is essential to reflect on how to teach the content.

This article describes the results of a study carried out with a group of elementary school teachers on the teaching of Plane Geometry in elementary school, using the Educational Design Research (EDR) methodological approach. The research problem that motivated this work was defined as: How can photographic images contribute to the teaching of Plane Geometry concepts? Based on this question, the central objective of the research was defined, which is to investigate how photographic images foster the construction of Plane Geometry concepts with teachers who teach mathematics in basic education. The study problem was identified after an interview with teachers who taught mathematics to students in the final years of elementary school. They were asked about a problem that distressed them, i.e. some content that they found most difficult to work on with their students. The answer was unanimous: "Plane Geometry content", and they explained that it was very abstract. They reported that they had difficulties locating themselves in the plane and in space, as well as using technology in their classes, visualizing geometric objects, establishing connections between mathematical content and everyday life and relating what was being worked on to the social context and the student's reality. Once the problem had been defined, we went on to discuss a proposal to build an educational product that would be meaningful to the group of teachers for future work with their students.

With this in mind, the idea was to create an educational product with the characteristics of didactic curricular material, in other words, a didactic sequence of activities that would enhance the learning of these teachers and, at the same time, contribute to their classroom work with their students. The proposal was therefore to build a teaching and learning sequence based on photographic images taken during the "Math Trail" dynamic in the streets of the city of Santa Cruz do Sul – RS, Brazil. Starting with a previously defined route, the participants were instructed to take photographic images of objects that could involve geometric concepts. The collection of images was the starting point for creating problems and building concepts of Plane Geometry. According to Vale and Barbosa (2015), Mathematical Trails promote concrete learning experiences, creating an atmosphere of adventure and exploration and enhancing the development of problem solving and formulation, communication, establishing connections and applying mathematical concepts to real situations. Using this dynamic, teachers "can learn mathematics in the environment and see its applicability" (Vale & Barbosa, 2015, p. 331). Therefore, the participating math teachers were instructed to observe the streets, buildings, traffic signs, gardens and squares and to photograph everything they thought they could relate to mathematics.

In addition, we chose EDR to address the problem, as it is a suitable research genre to address complex educational practice, such as geometry in primary school. According to Kneubil and Pietrocola (2017), EDR refers to a new, interventionist methodological approach that aims to combine theoretical and practical aspects of educational research. The authors argue that, in the field of science education research, it has been used to plan, implement, and evaluate teaching-learning sequences that address specific content.

2.0 Theoretical Foundations

In order to understand the importance of the proposed problem, we investigated some factors that allowed us to establish that this was, in fact, a systemic problem[1], and not just for this group. Initially, we searched for information in the PISA documents, known in Brazil as the Program for International Student Assessment, as well as information contained in the reports provided by the Ministry of Education (MEC) on students' proficiency levels. Based on these elements, we researched the results of the average performance in mathematics and the proficiency levels of participating students in Brazil and the Organization for Economic Cooperation and Development (OECD), as well as the performance of Brazilian students in the content category, Space and Shape, of the mathematics subject.

The research highlighted that of the 78 countries participating in PISA 2018, in the world ranking in mathematics, Brazil appears in the last places, occupying the 70th position. In this assessment, students are divided into 6 (six) levels of proficiency. The percentage of students in each country who reach good levels, such as proficiency levels 6, 5 or

[1] *Systemic* are all the necessary interconnections (Capra, 1997) that allow us to see the system as a whole, in a broader way.

4, indicates how well these countries manage to foster sublimity in their education systems. Among the four categories of mathematical content (Variations and Relationships; Space and Shape; Quantity; and Uncertainty and Data) that were assessed, the area with the most critical performance was Space and Shape, which involves geometry, in which 70.9% of the Brazilian students assessed are at level 1 and below in this category. As such, according to the MEC, the Space and Shape category is considered the lowest in the PISA performance index.

After analyzing the performance that Brazilian students have been showing, specifically in geometry, we checked whether these contents are being covered in the basic education curriculum. To do this, we had to check what the National Curriculum Parameters (PCNs) (Brasil, 1998) and the National Common Curriculum Base (BNCC) (Brasil, 2017) say about this category and this content. In the PCNs (Brasil, 1998), the content category includes the name Space and Shape, which highlights classical, axiomatic geometry and its internal relationships, and also "presupposes that the mathematics teacher explores situations in which some geometric constructions with rulers and compasses are necessary, such as visualization and application of properties of figures, in addition to the construction of other relationships" (Brasil, 1998, p. 51).

In Brazil, the document developed to guide teaching, the current BNCC, promulgated by the Ministry of Education (MEC) in 2017, specifies, in the area of mathematics in the final years of elementary school, that the contents relating to classical geometry are still present. Furthermore, in the area of mathematics in the final years of elementary school, the BNCC (2017) states that pedagogical decisions should be oriented towards the development of "competences and skills" (Brasil, 2017, p. 15). These guidelines are in compliance with an international trend coming from countries that performed better in the 2018 International Student Assessment Program (Brasil, 2018), because they have effective national guidelines for designing their curricula and refer to the development of competencies, such as South Korea and Finland.

Thus, in the context of the BNCC, the idea of competence is used in the sense of the application of school knowledge, understood broadly, such as concepts, procedures, values and attitudes. In this way, being competent means being able, when faced with a situation or problem, to make connections and use the knowledge built up.

In Brazil, D'Ambrósio (2004) describes that the teaching of mathematics should be based on the relationship with everyday life, contextualized, making the subject capable of apprehending, understanding, explaining and dealing critically with new situations and the reality that surrounds them. In this sense, the aim is for teaching to be geared towards giving students greater mastery and cognitive ability to make social use of their mathematical knowledge, both inside and outside school.

This perspective allows us to say that mathematics is much more present in our daily lives than we realize. According to Lorenzato (1995), geometry is part of our daily lives. In view of this, even though we know that in teaching the category of Space and Shape, i.e. geometry, is so essential, Almouloud et al. (2004) state that Space and Shape is one of the categories of mathematical content that receives the least attention in class. The authors justify this because of the difficulties identified in our education system, which defines education policy "with general recommendations and guidelines on methods, content and how to do things, leaving each school to define the content it deems important for the education of its students, which means that geometry is often forgotten" (Almouloud et al., 2004, p. 99). In this case, it is pointed out that this neglect of geometry is also linked to the precariousness of teaching related to teacher training.

> We can point out, in relation to teacher training, that it is very precarious when it comes to geometry, since initial training courses do not help them to reflect more deeply on the teaching and learning of this area of mathematics. On the other hand, continuing education still doesn't meet the expected objectives in relation to geometry (Almouloud et al., 2004, ibid.) (our translation).

As a result, it is worth pointing out that Brazil has a large territory and it is difficult to define and implement a curriculum that is national and unique for all educational establishments. This situation is recognized by the Law of Guidelines and Bases (LDB No. 9394/96), which gives education systems the autonomy to define their own curricular proposals, as long as they are in line with federal regulations. Faced with this description and the need to put curricula into effect, the education systems have been drawing up their own proposals and curricular guidelines.

According to Pavanello (1989), the difficulties pointed out in relation to curriculum organization, together with teachers' lack of preparation in working with geometric concepts, especially in basic education, mean that they fail to comply with the subject's syllabus. The author states that:

> [...] Among those teachers who included geometry among the topics to be developed in the classroom, many said that, due to lack of time, they couldn't even partially cover it. When asked about the time of year set aside for this subject, they invariably replied that it was the last term, or at best the last quarter of the school year, which seemed to indicate that, consciously or not, lack of time was being used as an excuse for not working with geometry. (Pavanello, 1989, p. 6) (our translation).

Despite this, Lorenzato (1995) points out that there are numerous causes for this neglect and abandonment of geometry in Brazilian schools, pointing out two of them that are directly linked to the classroom. The first reinforces "that many teachers do not have the geometric knowledge necessary to carry out their pedagogical practices" (Lorenzato, 2015, p. 3) and, above all, considers that "the teacher who does not know geometry also does not know the power, beauty and importance that it has for the formation of the future citizen, so everything indicates that, for these teachers, the dilemma is to try to teach geometry without knowing it or else not to teach it" (Lorenzato, ibid.). The second cause of the omission of geometry "is due to the exaggerated importance that the textbook plays among us, either because of the poor training of our teachers or because of the exhausting working hours to which they are subjected" (Lorenzato, ibid.).

Pavanello (1989, p. 9) investigated the relationship between the failure of geometry teaching and teacher unpreparedness, stating that, "although this teaching has proved problematic not only in Brazil, but also in the rest of the world – so that it has been the subject of much debate and research, this research, it seems to me, focuses on 'how' to teach this content". According to the author, the central points that point to the decline of geometry in school are the problems with teacher training and the omission of geometry from textbooks. Pavanello (1989) also stresses that these problems have persisted until the new generations and these causes certainly contribute to the failure, neglect and abandonment of geometry and, consequently, to the low performance of students.

> All these issues and others that have followed over the years have generated controversy and helped to highlight the absence of geometry in schools, the repercussions of which today interfere with the knowledge of current teachers. Therefore, it can be said that content that has not been learned by teachers will not even be transmitted, let alone interacted with – creating a vicious circle – which consequently affects generations of students who do not learn geometry (Pereira, 2001, p. 7) (our translation).

In Pavanello's (1989) work, the author adds that in order to improve the teaching of geometry, it is necessary to invest in new studies, including successful teaching and learning experiences with students and in teacher training. According to the author, it is also essential to develop materials that encourage the use of geometry and to train teachers to use them.

The search for alternatives to qualify work in this area has intensified in recent times. We would especially like to highlight the studies by Santos and Nacarato (2014) and Flores (2007), which showed that images and drawings of geometric figures are essential for the development of geometric concepts. In addition, we would highlight research by authors who have investigated and discussed questions about connections with images linked to the teaching of mathematics, reinforced in the studies by Flores (2007), Feldman-Bianco and Leite (1998), Martins and Tourinho (2013), Santaella (2012) and Manguel (2006). These works are the result of research that refers to the study of images as an object in the academic field, as well as to visual perception and thinking, which aims to link images to the teaching of mathematics, presenting the difference in using them in the classroom, specifically as an artifact.

According to Giroux and McLaren (1995), the world is totally dominated and surrounded by images, especially those generated by technology. When you look at the television, a magazine, a book, go to the movies, access the internet on your cell phone or computer, or see a billboard, you experience, inside and outside your home, immersion in the world of images.

> Increasingly, we are seeing this expansion of images, mainly due to the exacerbated use of social networks, where users share a multitude of images, including photographs, videos, cartoons and drawings. In this scenario, we understand how necessary this reflection on the image is, both in terms of unveiling the representation of the world that it carries, and in terms of the potential it has to guide human thought and action (Carlos, 2012, pp. 222-223) (our translation).

In this context, we agree with Santaela (2012) when he says that schools need to be transformed in order to enable the presence and proper use of these resources, so that students can interact with these media and educate themselves. In this sense, the choice was made to use images taken in the environment in which the research participants live, because it is understood that this can help in the process of learning content, especially geometry, as well as making it possible to understand their reality.

According to Martins and Tourinho (2013, p. 90), images can be used in schools because "[...] they make it possible to incorporate issues that have been outside the area of interest of school education, especially the effects that forms of education have on the construction of the subjectivity of boys, girls, young people and adults".

In this sense, teaching mathematics using images can help us understand that they are not only used to inform and illustrate, but also to educate and produce knowledge, as Manguel (2006, p. 21) states when he points out that "images, like stories, inform us" and can therefore become an "excellent teaching resource" designed to arouse students' interest.

"Reading" images enables students to learn about mathematical fundamentals in another language, in which mathematical concepts, procedures and representations are naturally identified. According to Kossoy (2001, p. 153), "[...] the image informs about the world and life, but in its own expression and aesthetics, because there is a plastic thought, just as there is a mathematical thought or a political thought".

Referring to the pedagogical issue of teaching mathematics, the image, in general, and "photography, in particular, has played an increasingly important role in dealing with the contents of this discipline" Kossoy (2001, p. 153). The author also states "the fact that the photographic image can perform various functions, including illustrative, communicative, decorative and epistemic, and these functions will contribute pedagogically to the teaching and learning process of the subject".

The use of photographic images as a didactic resource in math classes is useful and can help explore various concepts. According to Cavalcante et al. (2014):

> As a teaching tool, photography is useful for explaining, exemplifying, raising awareness, provoking doubts and questions. In this way, it can be used as a teaching methodology in different ways in different areas of study, depending on

> the needs of each teacher (Cavalcante et al., 2014, p. 11) (our translation).

In this sense, Carlos (2012, p. 260) believes that the use of other languages, such as images, "[...] can resize and revitalize the teaching-learning process, improve students' self-esteem, reduce school dropouts and challenge educators to promote other pedagogical activities".

In addition, people better understand what is visualized, as they often have great difficulty assimilating abstract or even surreal objects.

> Photography and drawing allow meanings to be penetrated through spatial memory and the association of images. The exercise of analyzing images stimulates visual perception and makes one accustomed to seeing an X-ray with suggestions of invisible meanings that go beyond the framework of two dimensions (Feldman-Bianco & Leite, 1998, p. 43) (our translation).

Images with a greater density of information bring up the need to think and analyze descriptive possibilities. In this way, Martins and Tourinho (2013, p. 92) state that working with images in the classroom "makes it possible to build exciting projects so that everyone (with different differences and positions) can find their place to learn meaningfully and build experiences of knowledge that allow them not only to interpret the world, but also to act in it".

The text of the PCNs (Brasil, 1998) recommends the use of graphical information in the teaching of mathematics as an indispensable pedagogical resource in any learning process:

> Reading graphical information in mathematics is an important aspect, as it helps to understand concepts and develop graphic expression skills. The availability of modern resources for producing images imposes the need to update mathematical images in accordance with technological and artistic trends, incorporating color, graphics and photography, as well as the importance of teaching students how to use these resources (Brasil, 1998, pp. 45-46). (our translation).

Our study is aimed at analyzing the contributions that an educational product, built from photographic images, can provide to elementary school mathematics teachers, in continuing education, in the construction of geometry concepts.

3.0 Study and methodology

In this study, we adopted the EDR approach as a research option in order to respond to the purposes of the study. Among the different conceptions of EDR that exist, we adopted the one created by McKenney and Reeves (2019), Figure 1, with three main phases: (phase 1) analysis and exploration, (phase 2) design and construction and (phase 3) evaluation and reflection, which guide the understanding and development of the intervention.

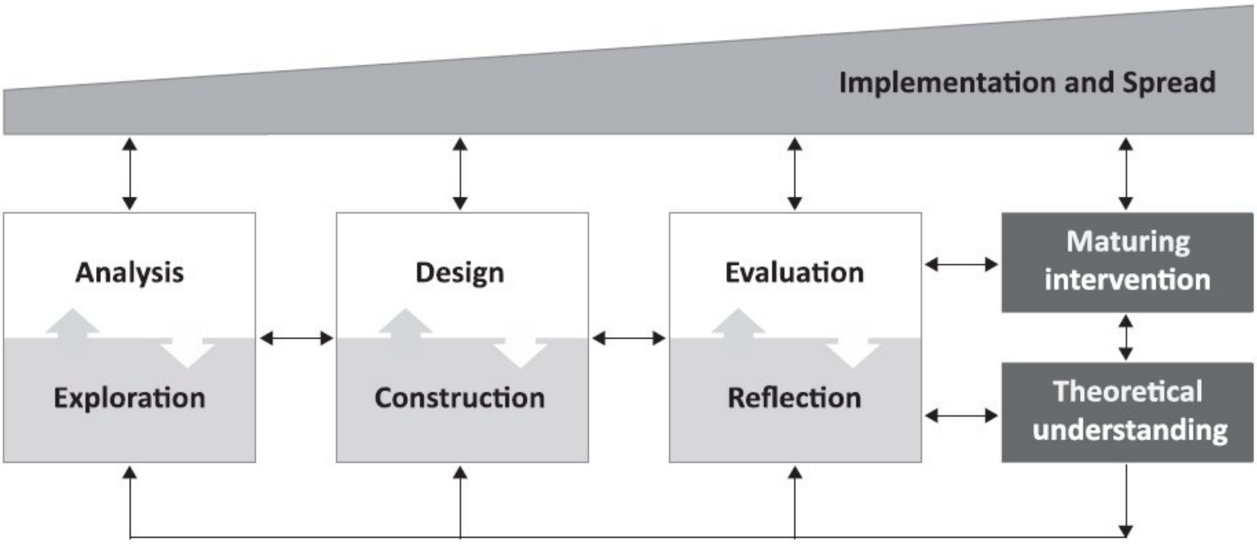

Figure 1. Model for conducting educational design research, suggested by McKenney & Reeves (2020, p. 86)

Each of these three phases "involves interaction with practice and contributes, directly or indirectly, to the production of theoretical knowledge and the development of an intervention, which matures over time" (McKenney & Reeves, 2019, pp. 83-84). The authors explain that, in Figure 1, the arrows indicate that the process is iterative and flexible. Iterative because the results of some elements feed into others repeatedly and flexibly because, although some general flow is indicated, many paths can be taken.

According to McKenney and Reeves (2019, p. 6), Educational Design Research "is a genre of research in which the iterative development of practical solutions to complex educational problems also provides the context for empirical investigations that generate theoretical understandings that can inform the work of others". The authors also clarify that as the iterative development of practical solutions to complex educational problems takes place, theoretical understandings emerge, and that these two outcomes (development of practical solutions and theoretical compressions) are considered one of the pillars of this genre of research. In addition, EDR presents, according to Plomp (2013), systematic studies in order to design and develop educational interventions as a solution to a real-life problem.

Design studies are considered to arise from problems that teachers and/or students face in real educational contexts. In this sense, according to Reeves (2006), an attempt is made to solve these problems by constructing appropriate design principles, which, however, are developed during the design process.

Design research protocols require long-term collaboration involving researchers and practitioners. It integrates the development of solutions to practical problems in learning environments with the identification of reusable design principles (Reeves, 2006, p. 52). As such, it provides an improvement in educational practice and the creation of a new product-based theory, because by contributing to theory, EDR presents a practical product.

Regarding the participants, the research relied on the contributions and assistance of two collaborative groups. The first collaborative group represents the knowledge of practice, made up of 3 (three) math teachers who work with elementary school students and teach in different schools in the municipal education network in the municipality of Santa Cruz do Sul – RS, Brazil. The second collaborative group represents academic-scientific knowledge, which we call collaborators, made up of teachers, academics, doctoral students and researchers from the Postgraduate Program in Mathematics and Science Teaching at the Franciscan University of Santa Maria – RS (UFN), who contributed to improving the educational product.

Thus, with the meeting of these two types of knowledge (academic-scientific knowledge and practical knowledge), the educational product was produced, applied, refined and improved. It should be noted that, in order to preserve their identities, the three participating teachers have been anonymized by their letters (PA, PB and PC).

Below we describe the activities carried out in each of the three main phases of the process carried out in this study.

3.1 Phase 1 - Analysis and Exploration

Initial phase 1 "includes the analysis and exploration of the problem in question" according to McKenney & Reeves (2019, p. 80). It was carried out during the researcher's first visit to the teachers of an elementary school who, in an interview, reported a problem with the teaching of geometry that distressed them. Based on this problem, a didactic sequence of activities on Plane Geometry at elementary school level was developed, so that it could contribute and help these teachers overcome their difficulties in teaching Plane Geometry. This meeting made it possible for the participants to feel part of the project and, at the same time, part of the construction of the activities based on the real context through collaborative work.

As EDR is a methodological approach that seeks to solve complex problems arising from real environments, we decided to bring the group of participating teachers together, motivate them to seek out information and get to know a little more about the local culture, record images from their perspective and establish connections with mathematics, using the "Math Trail" dynamic. According to Vale & Barbosa (2015 apud Cross, 1997), a Math Trail is a sequence of stops along a pre-planned route through which students can learn mathematics in their surroundings and see its applicability. During this journey, the participants visited the São João Batista Cathedral, which is the largest Gothic-style cathedral in Latin America and has a rich architecture, in which the participants were challenged to collect photographic images that caught their attention or that they considered important to record and that served as a didactic resource for the elaboration of the educational product.

3.2 Digital realisation of the design cycles

In the second phase, "design and construction", we carried out the design, which "resembles creation" (McKenney & Reeves, 2019, p. 85), i.e. we designed the initial version of the prototype and carried out a micro-cycle of evaluation. The feedback from the group of collaborators was fundamental so that we could make corrections and improvements to the prototype. According to McKenney and Reeves (2019, p. 175), "expert evaluation is also used to collect ideas for improvement".

Some preliminary principles were established according to the analysis and exploration phase, especially based on the theoretical investigations conducted regarding the problem at hand. Table 1 illustrates each of them and their origin.

Table 1: Theoretical basis and preliminary design principles

Theoretical Basis	***Design principles***
Giroux & McLaren (1995); Flores (2007); Feldman-Bianco & Leite (1998); Martins &Tourinho (2013); Santaella (2012) & Manguel (2006); Santos & Nacarato (2014).	a) the use of photographic images and technological resources facilitates the understanding of geometric concepts;
Kossoy (2001); Cavalcante et al. (2014); Cifuentes, J. C. (2005); Nascimento, Benutti & Neves (2007).	b) linking art and mathematics enhances visualization and facilitates the exploration of the geometric properties of flat figures;
D'Ambrósio (2004); McKenney & Reeves (2019); Vale & Barbosa (2015); Cross (1997).	c) activities designed to take into account the local reality and its cultural aspects are motivating for teaching geometry.

Source: Table prepared by the author, 2024.

Therefore, the creation of the educational product was based on the following preliminary principles:

a) the use of photographic images and the use of technological resources facilitate the understanding of geometric concepts;

b) linking art and mathematics enhances visualization and facilitates the exploration of the geometric properties of flat figures;

c) activities designed to take into account the local reality and its cultural aspects are motivating for teaching geometry.

The educational product's activities were built collaboratively and involved Plane Geometry content, created from the photographic images recorded in the "Mathematical Trail" dynamic and following the defined design principles. Activities were proposed that explored the concepts of polygons and their properties, perimeter and area of flat figures, the study of geometric transformations in the plane, such as reflection, translation and rotation, notions of perspective, parallel and perpendicular lines. After this first micro-cycle, we applied the activities with the participating teachers.

3.2.1 Application of the didactic sequence of activities

In this section we present some of the activities that made up the educational product, using the images captured on the Math Trail as a starting point. The contents of Plane Geometry that could be explored collaboratively with the teachers taking part in the research were delimited. Activity 1.1 consisted of producing a video about the history of St. John the Baptist Cathedral. The video can be accessed at: https://youtu.be/sMHcBN4nTKc and contains the narration, photographic images and footage taken by the researcher and the participants during the visit. The teachers' narratives show the impact of the dynamics.

PC: *"This was an incredible experience, one I'll never forget. At first, I was apprehensive because I didn't know what this meeting would be like. I was delighted with everything, with the dynamics, with the Cathedral. It was fantastic! It was so worth it!"*

PA: *"I loved this meeting. I never imagined doing a dynamic like this. I kept imagining my students here. It would be an incredible lesson, with many contributions, both mathematical and cultural."*

PB: *"I'll tell you something, I thought it was great! From the start in the square, then we walked all the way observing, until we arrived at the Cathedral. Wonderful! I'm delighted with this format and with what you've given us"*.

Activities 1.2 and 1.3 then focused on investigating the geometric polygons present in the composition of the paving stones on the front of the Cathedral, identifying the properties of geometric figures, the concepts of perimeter and area of polygons, recognizing regular polygons and checking the sum of the measures of the internal angles of regular polygons, concepts of rotation, translation and the use of GeoGebra software.

Activity 1.2

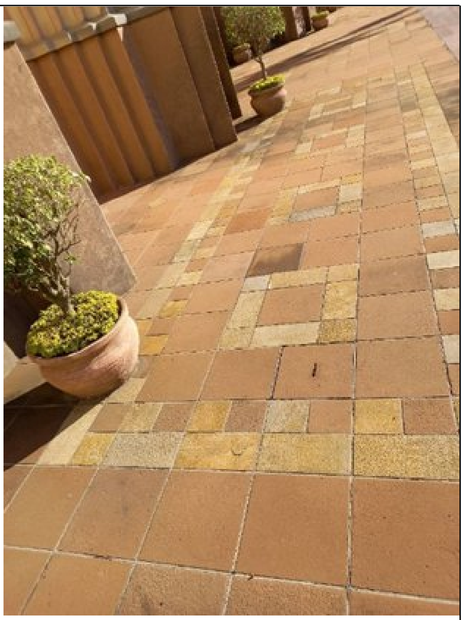

Figure 2. Photogtaphy of PA, 2021

During the construction of the Cathedral, the entire external sidewalk was covered with sandstone. According to the technical team working on the project, this material, due to its porosity, allows moisture from the foundations to escape by capillarity and evaporation, thus avoiding dampness, which always leads to stains.

The paving is made up of regular, uniform stones, with the darker tones positioned side by side. On the other hand, the apparently lighter tones stand out as geometric details on the floor.

Look at the geometric design formed on the sidewalk (Figure 2) and identify which distinct geometric figures you can see. Describe their properties.

What do you think is the difference between a square and a rectangle? Is a square a rectangle? Is a rectangle a square? Justify.

Activity 1.3

Look at Figure 3, which represents a paving slab. Which geometric figures make up the image? Describe them.

Figure 3: Photography from the PA participant

a) Highlight the central part of the tile (as shown in the image) and assign a value to each of the sides of the highlighted figure and calculate the sum of its sides. What does the result mean? Multiply the measurements of the perpendicular sides. What does the result mean?
b) Considering the highlighted geometric figure, how many internal angles does it have? What is the measure of each angle? What is the name of each of these angles?

In solving activities 1.2 and 1.3, the ability to think geometrically and establish relationships and comparisons was evident, as the teachers demonstrated their ability to visualize, connect concepts, draw and carry out operations, according to the dialogue established between them when describing the properties of the figure.

PC: *"The square has four sides, which are two by two parallel to each other".*

PA: *"Four equal sides. The four sides have the same length. In a square, each side is the same length, unlike a rectangle."*

PB: *"Four equal angles, they are right angles, they measure 90° each".*

PC: *"The diagonals. The division by the diagonals forms triangles.*

PA: *"The diagonals of a square are lines that intersect at the square's midpoint, which forms four triangles."*

PC: *"The triangle that forms with a diagonal is a right triangle."*

It was possible to see that the participants identified some basic properties during the dialog, but even if they didn't mention all the properties, they made some correct associations. It was also noted that the participants had difficulties in using mathematical language properly, specifically when asked to describe the properties of the geometric objects observed. When recognizing a geometric object, it is essential to differentiate one from another by using its characteristics and properties and, therefore, using the correct language is essential in the process of constructing concepts. In this respect, despite correctly expressing the concept of perimeter (the sum of the sides of a polygon), they had difficulties when asked about the connection between perimeter and area.

Researcher: *"What happens to the perimeter of a regular polygon if the measurements of the sides are changed while maintaining the same area?"*

PA: *"I believe that the perimeter will increase or decrease in value if the measurements of the sides are increased or decreased."*

The other participants did not express their ideas and argued that they had never thought about it. After proposing different situations in which the measurements of the sides of a quadrilateral were varied, the participants understood the question posed by the researcher. Thus, it is not enough to present the definition of a concept in a formal way, but it is necessary for it to be understood in different situations, along with the use of clear language. In this sense, we agree with Barguil (2016, p. 236) when he states that "it is essential to know and use the appropriate language in the teaching and learning process in mathematics, as it is a basic element for communication between teacher and student as well as for understanding the information established".

Activity 1.4 Analyzing the composition of the Cathedral's rose window

As you enter the Cathedral, you can see a large rose window above the main entrance door. The rose window is an architectural element whose decoration is built in a radial direction, like the petals of a flower. Formed from a geometric design, filled with colored glass, which fills a structure formed from stones, the so-called tracery, it can be seen from both the outside and inside of the Cathedral. Its position allows more light in, favoring a radiant effect through the combination of several shades of the same color that illuminates the internal aisle towards the altar.

Figure 4. Rosacea viewed from the inside of the Cathedral

Figure 4 shows a photo of the rose window from the Cathedral's altar. The rose window has structural lines, which become darker when contrasted with the colored luminosity of the stained glass windows, which are diluted by the darker interior walls. It is precisely in this dilution that the luminosity with the mixtures of tones radiates towards the altar. In the photograph, Figure 5, below, the Cathedral's rose window stands out for its geometric features, giving rise to geometric shapes.

Figure 5. Rosacea viewed from the inside of the Cathedral

a) Which geometric figures are present in the composition of the rose window? Describe them.

b) In the center of the rose window is the monogram "IHS", which is an abbreviation of the Latin phrase "Iesus

Hominum Salvador" (Jesus Savior of Men). From this center, the geometric lines begin to give rise to the representation of the petals of the Rosacea.
If you look closely at the geometric shapes that make it up, you can see circles divided into equal parts. Can you explain how we can divide a circle?

b1) in two equal parts?
b2) in three equal parts?
b3) in four equal parts?
b4) in five equal parts?
b5) into six equal parts?

Describe the process of dividing a circle into equal parts.

c) Examine the rose window in Figure 5 and construct a replica using pencil, paper, ruler and compass or GeoGebra.
d) Use your creativity and come up with a new rosette model.

According to the Houaiss online dictionary (2019), a rose is defined as a symmetrical figure formed from a circle. It is an ornamentation in the shape of a rose, composed of a center in which the petals are arranged in a circle. During the process of constructing a new rose proposal, participant PB, referring to future work with her students, pointed out that:

PB: "*the student is going to think about everything that was asked before in order to draw it. They'll think about the larger circumference, the smaller circumference. To make the petals, they will divide them into 6 equal parts and then divide them again. They will use their creativity. The student will think about everything that was asked before in order to build this replica. They'll use math and art to build it.*"

The participant's speech allows us to conclude that she understood the connection that can be established between geometry and art. This perception is in line with what Nascimento, Benutti and Neves (2007, p. 2) say, when they point out that "the rose window becomes an important element in the educational process, opening up possibilities for exploring various concepts linked to graphic representation", in this case, circumference, radius, diameter, midpoint, bisector, division of the circumference into equal parts.

Figure 6 below shows the rosette proposal built by each participant for item *e*.

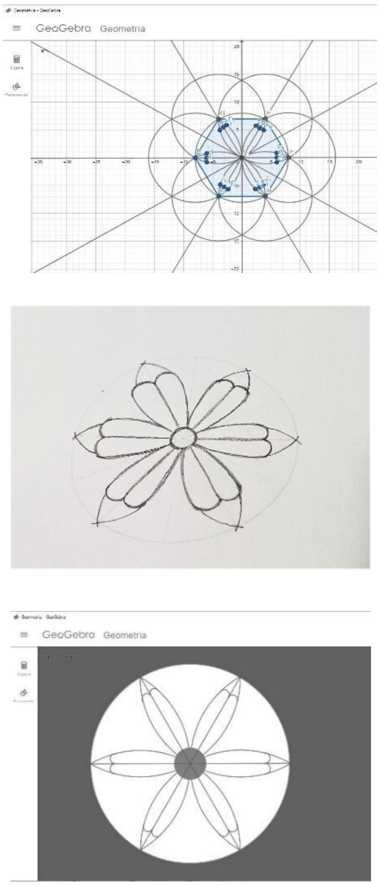

Figure 6. Replica of the rose window built by the participants

The activities described are just examples of a larger set that involved the study of other Plane Geometry concepts. Details of the educational product can be found at: https://www.ufn.edu.br/site/ensino/mestrado/programa-de-posgraduacao-em-ensino-de-ciencias-e-matematica.

3.3 Phase 3: Evaluation and Reflection

The evaluation was based on the objectives and preliminary design principles initially outlined. It was carried out throughout the process of developing the activities and was examined in terms of validity, practicality and effectiveness, according to Nieveen (1999).
The evaluation of the educational product was carried out throughout the process of developing the pedagogical intervention and its validity was examined by looking at the preliminary design principles, the selection of content, together with the literature review. First of all, the selection of content was based on the participants' statements of their needs, as revealed in the interviews conducted at the first meeting. The proposition of the activities, the literature review and the establishment of the preliminary design principles had the support of the group of university collaborators who accompanied the entire process of preparing the educational product and its implementation.

Throughout the preparation and implementation period, the group of collaborators helped to analyze the participants' difficulties, suggesting changes and improvements to the educational product and the necessary readings to better understand the difficulties that arose.

To check practicality, we analyzed the opinions of the teachers taking part in the research and the group of collaborators, recorded in the researcher's diary. All the participants were unanimous in stating that the activities are proposed and constructed together with them, and based on their photographic work, that the intervention is attractive and perfectly usable in the future with their students. In addition, the language used and the way the questions were posed did not make it difficult for the participants to understand the concepts involved. This can be seen in the participants' statements:

PC: "*It's very good to work with photographic images in this way. We understand better, it makes a difference to learning and teaching*".

PB: "*I think these activities will make my students appreciate mathematics more*".

PA: "*I loved the video, I thought it was really cool*".

PB: "*The activities were very interesting, I think they will attract the attention of those who develop them. Concerns: quantity and time, which is why I think it would be interesting to present them to the group of participating primary school teachers and ask them, after carrying them out, to comment on these two factors in relation to their students. In this way, it will be possible to reformulate or reorganize the activities*."

PC: "*First of all, congratulations on the activities! I think the organization of the activities is great! That's what I expected from this work*."

According to Van den Akker (1999, p. 10), "effectiveness refers to the degree to which the experiences and results of the intervention are consistent with the intended objectives". In our case, it is understood that effectiveness was also achieved because it was possible to see from the participants' answers above that the activities met their expectations.

In terms of what this experience was intended to develop, the data showed that the design principles initially proposed were appropriate and did indeed contribute to the educational product organization. This can be seen in the participants' statements. For example, participant PB said:

PB: *"This training experience allowed me to relate mathematics to the context of the city and, at the same time, learn more about the art expressed in the images in the Cathedral."*

Participant PC put it this way:

PC: "*It was very good. I liked the idea of exploring João Batista Cathedral and bringing the subject into the classroom. The walk* *through the city streets also allowed me to be more observant about how to relate mathematics to the place where we live."*

Similarly, PC put it this way:

PC: "*Throughout the work I took advantage of this and worked on one of the activities, which dealt with symmetry with my eighth grade students. They were delighted with the explanation about the Cathedral. The activities are captivating. Geometry became easier to work with."*

It was found that throughout the implementation process, the participants deepened their skills related to the teaching of geometry and, in addition, related to the use of a technological tool, especially GeoGebra, as evidenced in the speech of participant PC:

PC: *"It's interesting to study math from problems in our daily lives and it was nice to use GeoGebra instead of the ruler and compass. I liked studying math this way."*

The testimonies indicate that the use of images and technological tools such as GeoGebra can arouse interest and enable participants to get involved in constructing the solutions to the activities. It can also be inferred that the use of computer programs associated with images facilitates the study of geometry, arouses enthusiasm and motivation to carry out the activities.

The experiment also enabled the participants to propose activities based on their own material. It turned out that the teachers generally followed the ideas contained in the textbooks, repeating what they proposed. The fact that they took part in the collaborative group enabled them to create problems based on the material captured on the Math Trail.

PB: "*The fact that it was collaborative work enabled me to ask questions that turned into problems. That's the learning I take away from this work."*

The book is excellent teaching material and helps teachers in their classroom work, but we need to go further, in other words, to give teachers the opportunity to create new situations.

Thus, the experience developed was not limited to contributing only to the teaching of Plane Geometry, but also created opportunities for them to experiment with the creation of problems other than those contained in textbooks and the incorporation of technology in the solution of activities. Therefore, during the course of the work, it was possible to validate the proposed design principles, as the teachers' narratives show.

Table 2 below summarizes the design principles, which authors underpinned the work and which design strategies were used.

Table 2: Design principles and design strategies

Design principles	**Theoretical Basis**	***Design strategies***

a) the use of photographic images and technological resources facilitates the understanding of geometric concepts;	Giroux & McLaren (1995); Flores (2007); Feldman-Bianco & Leite (1998); Martins & Tourinho (2013); Santaella (2012) & Manguel (2006); Santos and Nacarato (2014).	A Math Trail through the streets of the city, including Santa Cruz do Sul Cathedral.
b) linking art and mathematics enhances visualization and facilitates the exploration of the geometric properties of flat figures;	Kossoy (2001); Cavalcante et al. (2014); Cifuentes, J. C. (2005); Nascimento, Benutti & Neves (2007).	Use of images that express the art in the Cathedral and captured on the Mathematical Trail.
c) activities designed to take into account the local reality and its cultural aspects are motivating for teaching geometry.	D'Ambrósio (2004); McKenney & Reeves (2019); Vale & Barbosa (2015); Cross (1997).	Proposing problem situations based on images related to the city context.

Source: Table prepared by the author, 2024.

At the end of the implementation of the educational product, it was possible to re-evaluate all the activities and this moment consisted of taking a new look at the proposal that was initially built, in other words, the redesign of the educational product was carried out, the purpose of which was to adapt the language so that it could be used with students in the schools where the participants work. This stage involved all the participants in the research, including the group of collaborators from the university. The possible difficulties and obstacles that could arise in the classroom were analyzed, taking into account the content, the students' learning and the teacher's difficulties.

4.0 Final Considerations

This study reports the results of a research that aimed to analyze the contributions that an educational product, built from photographic images, can provide to elementary school mathematics teachers, in continuing education, in the construction of geometry concepts. Based on the definition of the problem, the objectives of the study were established, along with the choice of Educational Design Research as the methodological approach and the preliminary design principles that were validated throughout the process. Three design principles were highlighted, namely: a) the use of photographic images and technological resources facilitates the understanding of geometric concepts; b) relating art and mathematics enhances visualization and facilitates the exploration of the geometric properties of flat figures; c) activities designed to take into account the local reality and its cultural aspects are motivating for the teaching of geometry.

From the perceptions revealed in the teachers' speeches and in the resolutions of the activities that made up the educational product, it was possible to infer the influence of the teaching experience provided to the teachers with the EDR and the possibilities for changes in the way they perceive their performance in the classroom.

In the design principles listed, aimed at the relationship between art and mathematics, the use of technological resources and the relationship of the work with aspects of local and cultural reality, we found several factors responsible for these changes, such as: motivation, interest, teacher participation, among others. Innovation was established as an element that made it possible to develop a new classroom system, with an approach to the content through the use of photographic images. We therefore believe that these factors have made it possible to revive a love of teaching and learning, awakening teacher protagonism. In particular, the design principles that served as the basis for the construction of the educational product highlighted the expectation that approaching content through the use of images facilitated classroom work.

From the analysis of the data collected, it emerged that the activities built collaboratively between the group of teachers and the researcher, with the support of the group of specialists, enabled the identification of the potential and possible weaknesses that may appear in the development of the work with the students in a next application cycle.

Finally, in view of the process of learning and teaching, we believe that the use of images, together with technological resources and the establishment of a relationship between art and mathematics, taking into account local cultural aspects, is a way of planning actions to improve the continuing education of teachers.

5.0 References

Almouloud, S. A., Manrique, A. L., Ferreira da Silva, M. J., & Mendonça Campos, T. M. (2004). A geometria no ensino fundamental: reflexões sobre uma experiência de formação envolvendo professores e alunos. *Revista Brasileira de Educação [online]. Rio de Janeiro*, *27*, 94–108.

Barguil, P. M. (2016). Geometria na Educação Infantil e no Ensino Fundamental: contribuições do Fiplan. In: Andrade, F. A, Tahim, A. P. V. O & Chaves, F. M. (Eds.). *Educação, saberes e práticas*. Curitiba: CRV, pp. 233–250.

Brasil (2017). Ministério da Educação. *Base Nacional Comum Curricular (BNCC). Educação é a Base*. Brasília, MEC/CONSED/UNDIME, (p. 600). Retrieved from: http://basenacionalcomum.mec.gov.br/images/BNCC_EI_EF_110518_versaofinal_site.pdf

Brasil (1998). Ministério da Educação. *Parâmetros Curriculares Nacionais (PCNs). Matemática: Terceiro e quarto ciclos do Ensino Fundamental*. Brasília: MEC/SEF. (p. 148). Retrieved from: http://portal.mec.gov.br/seb/arquivos/pdf/introducao.pdf

Brasil (2018). Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira (INEP). *Relatório Brasil no PISA 2018 (versão preliminar)*. Brasília-DF Inep/MEC. Retrieved from: https://download.inep.gov.br/acoes_internacionais/pisa/documentos/2019/relatorio_PISA_2018_preliminar.pdf

Capra, F. (1997). *The web of life – a new synthesis of mind and matter*. Editora Flamingo.

Cavalcante, J. S., Sousa, E. P., Garcia, N. R., Bezerra, C. S., & Silva, K. R. C. (2014). A fotografia como ferramenta no ensino de ecologia. In: *IV Simpósio Nacional de Ensino de Ciências e Tecnologia*. v. 14. Ponta Grossa/PR.

Carlos, E. J. (2012). *Educação e visualidade: a imagem como objeto do conhecimento*. Editora da UFPB, João Pessoa, (p. 288).

Cifuentes, J. C. (2005). Uma Via Estética de acesso ao conhecimento Matemático. *Boletim GEPEM (USU)*, 46, 55–72.

Cross, R. (1997). *Developing Math Trails*. Mathematics Teaching, 158, 38–39.

D'Ambrósio. (2004). Etnomatemática e educação. In: Knijinik. G., Wanderer, F., & Oliveira, C, J (Eds.). *Etnomatemática: currículo e formação de professores*. Edunisc. Santa Cruz do Sul.

Feldman-Bianco, B. & Leite, M. L. M. (1998). *Desafios da Imagem: Fotografia, icografia e vídeo nas ciências sociais*. Campinas, São Paulo: Editora Papirus.

Flores, C. R. (2007). *Cultura visual, visualidade, visualização matemática: balanço provisório, propostas cautelares*. Revista *ZETETIKÉ*, 18, Campinas: Unicamp – FE – CEMPEM.

Giroux, H. A. & McLaren, P. (1995). Por uma pedagogia crítica da representação. In: Silva, T. T. da, & Moreira, A. F. (Eds.). *Territórios*

Contestados: o currículo e os novos mapas políticos e culturais. Petrópolis: Vozes, pp. 144–158.

Houaiss, Eletrônico. (2019, June). *Instituto Antonio Houaiss*; Ed. Objetiva. Versão monousuário 3.0.

Kneubil, F. B. & Pietrocola, M. (2017). *A pesquisa baseada em Design: visão geral e contribuições para o ensino de ciências*. Revista Investigações em Ensino de Ciências, *22*(2), 1–16. https://doi.org/10.22600/1518-8795.ienci2017v22n2p01

Kossoy, B. (2001). *Fotografia & História*. *2*. ed. São Paulo: Ateliê Editorial.

Lorenzato, S. A. (2015). Como aprendemos e ensinamos geometria. In: Lorenzato, S. A (Ed.). *Aprender e ensinar geometria*. Campinas, São Paulo: Mercado das Letras.

Lorenzato, S. A. (1995). Por que não ensinar Geometria? *Revista da Sociedade Brasileira de Educação Matemática*, *3* (4), 3–13.

Manguel, A. (2006). *Lendo Imagens: uma história de amor e ódio*. São Paulo: Companhia das Letras.

Martins, R. & Tourinho, I. (Eds). (2013). *Processos & Práticas de Pesquisa em Cultura Visual & Educação*. Santa Maria: Editora da UFSM.

McKenney, S. & Reeves, T. C. (2020). Educational Design Research: Portraying, conducting, and enhancing productive Scholarship. *Medical Education*, *55*, 82–92. https://doi.org/10.1111/medu.14280

McKenney, S. & Reeves, T. C. (2019). Conducting Educational Design Research (2nd ed.). New York: Routledge.

Nascimento, R. A. do, Benutti, M. A., & Neves, A. F. (2007). Mandalas e rosáceas: em busca de novas abordagens para antigos conteúdos. In: Graphica 2007: desafio da era digital: ensino e tecnologia – *XII International Conference on Graphics Engineering for arts and Design; XVIII Simpósio Nacional de geometria descritiva e desenho técnico*. Anais Graphica 2007. v. 1. Curitiba: editora da UFPR. Retrieved from: http://www.degraf.ufpr.br/graphica2007/artigos/MANDALAS.pdf

Nieveen, N. (1999). Prototyping to reach product quality. In: Van den Akker, J., Branch, R. M., Gustafson, K., Nieveen, N., & Plomp, T. (Eds.), *Design Approaches and Tools in Education and Training* (pp. 125–136). Boston: Kluwer Academic.

Pavanello, R. M. (1989). *O abandono do ensino de geometria: uma abordagem histórica*. [Dissertação de mestrado em Educação, Universidade Estadual de Campinas]. Retrieved from: https://repositorio.unicamp.br/Acervo/Detalhe/45263

Pereira, M. R. de O. (2001). *A Geometria Escolar: Uma Análise dos Estudos Sobre o Abandono de seu Ensino*. [Dissertação de mestrado em Educação, Pontifícia Universidade Católica de São Paulo]. Retrieved from: https://tede2.pucsp.br/handle/handle/11182

Plomp, T. (2013). Educational Design Research: Part A: An Introduction. In: Plomp, T. & Nieveen, N. (Eds). *An Introduction to Educational Design Research* (pp. 9–36). Enschede: Netzodruk. Retrieved from: https://ris.utwente.nl/ws/portalfiles/portal/14472302/Introduction_20to_20education_20design_20research.pdf

Reeves, T. C. (2006). Design research from the technology perspective. In: Van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N. (Eds.). *Educational Design Research* (pp. 52–67). London, UK: Routledge. Retrieved from: https://www.researchgate.net/publication/285854675_Design_research_from_a_technology_perspective

Rodrigues, B. & Ponte, J. P. (2020). *Desenvolvimento do conhecimento didático de professores em Estatística: Uma experiência formativa*, *Zetetiké*, *22*(3), 138–167. https://doi.org/10.23925/1983-3156.2020v22i3p138-167

Santaella, L. (2012). *Leitura de Imagens*. São Paulo: Editora Melhoramento.

Santos, C. A. & Nacarato, A. M. (2014). *Aprendizagem em Geometria na educação básica: a fotografia e a escrita em sala de aula*. (1st ed.) Belo Horizonte: Autêntica Editora.

Vale, I. & Barbosa, A. (2015). Os Trilhos Matemáticos como contexto não formal de ensino e aprendizagem: uma experiência com futuros professores do ensino básico. *In*: *XXVI Seminário de investigação em Educação Matemática. Atas do XXVI Seminário de investigação em Educação Matemática. XXVI SIEM*, pp. 330–332, Lisboa. Retrieved from: https://www.researchgate.net/publication/335106471

Van den Akker, J. (1999). Principles and methods of development research. In: Van den Akker, J. et al. (Eds.). *Design Methodology and Developmental Research in Education and Training* (pp. 1–14). The Netherlands: Kluwer Academic Publishers. Retrieved from: https://link.springer.com/chapter/10.1007/978-94-011-4255-7_1

Van Driel, J., Bulte, A., & Verloop, N. (2005). The Conceptions of Chemistry Teachers about Teaching and Learning in the Context of a Curriculum Innovation. International Journal of Science Education, 27(3), 303–322. https://doi.org/10.1080/09500690412331314487

Author Profiles

Débora de Sales Fontoura da Silva Frantz is a Mathematics teacher and has a PhD in Science and Mathematics Teaching from the Franciscan University (2022), a Master's degree in Mathematics Teaching from the Federal University of Rio Grande do Sul (2015), a Lato Sensu postgraduate degree in School Guidance, Supervision and School Management from Centro Universitário Internacional (2018), Lato Sensu postgraduate degree in Mathematics Teaching with an emphasis on Physics, Statistics and Educational Informatics from the University of Santa Cruz do Sul (2010) and a degree in Mathematics Full Degree from the University of Santa Cruz do Sul (2008). She currently works as a substitute Mathematics teacher at the Instituto Federal Sul-rio-grandense, Câmpus Lajeado – RS and works as a Mathematics teacher at Colégio Marista São Luís de Santa Cruz do Sul. She has experience in the area of Mathematics, with an emphasis on Mathematics Teaching, working mainly on the themes: teaching and learning Mathematics.

Vanilde Bisognin she holds a full degree in Mathematics from the Federal University of Santa Maria (1971), a master's degree in Mathematics from the Federal University of Rio de Janeiro (1978) and a PhD in Mathematics from the Federal University of Rio de Janeiro (1992). She is currently a full teacher at the Franciscan University Center. Has experience in the area of Mathematics, with an emphasis on Mathematics Teaching, working mainly on the following topics: teaching and learning mathematics and mathematical modeling.

Author Details

Débora de Sales Fontoura da Silva Frantz
Postgraduate Program of Mathematics and Science Teaching
Franciscan University
Silva Jardim, 1175
Brazil
+5555 3025.9000
debora_frantz@hotmail.com (debora_frantz@hotmail.com)

Vanilde Bisognin
Postgraduate Program of Mathematics and Science Teaching
Franciscan University
Silva Jardim, 1175
Brazil
+5555 3025.9000
vanilde@ufn.edu.br

Editor Details

Prof. Dr. Tobias Jenert
Chair of Higher education and Educational Development
University of Paderborn
Warburgerstraße 100

Germany
+49 5251 60-2372
Tobias.Jenert@upb.de

Journal Details

EDeR – Educational Design Research
An International Journal for Design-Based Research in Education
ISSN: 2511-0667
uhh.de/EDeR
#EDeRJournal (our hashtag on social media services)

Published by

Hamburg Center for University Teaching and Learning (HUL)
University of Hamburg
Schlüterstraße 51
20146 Hamburg
Germany
+49 40 42838-9640
+49 40 42838-9650 (fax)
EDeR.HUL@uni-hamburg.de
hul.uni-hamburg.de

In collaboration with

Hamburg University Press
Verlag der Staats- und Universitätsbibliothek Hamburg –
Landesbetrieb
Von-Melle-Park 3
20146 Hamburg
Germany
+49 40 42838 7146
info.hup@sub.uni-hamburg.de
hup.sub.uni-hamburg.de