



Educational Design Research

Volume 8 | Issue 1 | 2024 | Article 63

Contribution Academic Article

Title **Design research on an online summer school in mathematics education: An insight into philosophical commitments**

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Abstract Design research in education pursues two goals simultaneously: the development of a design and a local theory that clarifies how the design can achieve the intended effects and under what conditions. The results are obtained in an iterative cyclical process including both the implementation of the design and the research on its implementation. Each cycle contains a prospective phase, in which the design is determined, and a reflective phase, in which the implementation of the design is examined. The nested connection of theory and design development leads to a reciprocal relationship between the two final products.

This article introduces an example of design research from mathematics education, in which an online summer school for doctoral students was to be framed in response to the sudden lockdown situation at the outbreak of the Covid-19 pandemic. At short notice, the original summer school was moved into the virtual space of a digital conference system resulting in new framework conditions; previous design principles were adapted and a rhythmical organization of space and time was theorized and implemented. Rhythm analyses show that design decisions to real-

Einige Autorinnen und Autoren dieses Special Issues sind Mitglieder des wissenschaftlichen Netzwerks Design-Based Research als methodologischer Rahmen in der Bildungsforschung (DBR-Netzwerk, gefördert durch die Deutsche Forschungsgemeinschaft (DFG) – 452077361).

ize ontological, epistemological and axiological commitments intertwined and thus fostered students' interactive learning processes. Consequently, the suggestion is made to merge the three commitments into a three-dimensional framework concept of ethico-onto-epistemological commitments. Such a framework concept would have the function of reconciling ethical, ontological and epistemological dimensions in design research in the pursuit of robust teaching-learning processes and incorporating the ethical responsibilities of design researchers conceptually for both design and local theory.

Keywords educational design research, ethico-onto-epistemological commitment, instrumental approach, online summer school, rhythm analysis, research pentagon

DOI dx.doi.org/10.15460/eder.8.1.2129

Citation Bikner-Ahsbabs, A. (2024). Design research on an online summer school in mathematics education: An insight into philosophical commitments. *EDeR – Educational Design Research*, 8(1), 1-26.

dx.doi.org/10.15460/eder.8.1.2129

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Design research on an online summer school in mathematics education: An insight into philosophical commitments

Angelika Bikner-Ahsbabs

1.0 Introduction

“Most educational research describes or evaluates education as it currently is. Some educational research analyzes education as it was. Design research, however, is about education as it could be or even as it should be.” (Bakker, 2018, p. 3)

This quote highlights the ontological function of educational design research, emphasizing its role in creating tangible impact on education. Design research in education is distinct because it takes both practical and scientific needs seriously when crafting reality to achieve an educational goal (Van der Wal, Bakker, Moes, & Drijvers, 2021). It is about investigating the implementation of the design to develop a local theory that explains how the design operates in specific contexts with particular individuals (Prediger, Gravemeijer, & Confrey, 2015). When developing a design, such as for teaching mathematics in schools, this is typically done in a cyclic manner. Each cycle consists of a prospective and a reflective phase (Bakker, 2018; Gravemeijer & Cobb, 2006). In the prospective phase, design choices anticipate the intended effect and what is necessary to design to achieve the intended effect. The implementation of these design choices in practice is then reflected upon, empirically investigated and theorized, potentially leading to a revision of the design with consequences for constructing a local theory. Such design processes are cyclic for two reasons: First, designers design for practical reasons, hence, how educational settings could or should be (Bakker, 2018, p. 3). They acknowledge that practice is complex (Campanella & Penuel, 2021; Prediger et al., 2015), in many cases too complex to gain the appropriate design in just one shot. Second, theorizing how the design works in practice comes from exploring the misfits between design and practice and guides the revision of the design for the next design experiment.

In 2020, we conducted a design research study shifting from a face-to-face setting to an online summer school as a response to the lockdown situation of the Covid-19 pandemic (Bikner-Ahsbabs, Trgalová, Maffia, Bakker, & Lembrér, 2022). It built on the tradition aimed at helping early career researchers in mathematics education develop their expertise in research by supporting their projects in various ways, writing and reading about research, presenting own research, taking and giving feedback, thus, communicating about research in groups. We adapted the topic-specific design research model as introduced by Prediger and Zwetschler (2013) to the topic of teaching and learning of conducting research. In this model, each cycle involves four steps:

(re-)structuring the topic to shape the teaching goal, designing the teaching-learning arrangement through design choices to achieve this goal, conducting a design experiment by research of the implementation of the design, and theorizing the results of the data analysis.

In research-based design, the research takes precedence to inform the design. In design research of the online summer school, designing and theorizing were not isolated processes; instead, they were interrelated, giving rise to both the design and the local theory as mutual manifestations focusing on the process rather than the products of learning (Prediger et al., 2015). Our philosophy of the learning processes focused on the social nature of learning, grounded in three specific philosophical commitments (adapted from Deane, 2018, and Patterson & Williams, 1998) that differentiate educational design research from non-transformative research (see Bakker, 2018):

1. The ontological commitment concerns the nature of reality and what really exists in relation to the participants involved, i.e., in our educational design research, the nature of human experience is considered emergent and constitutive, rather than deterministic or controllable (see Bakker, Angerer, Penuel, & Akkerman, forthcoming).
2. The epistemological commitment addresses the relationship between the knowers and what is known, i.e., in our educational design research, these knowers are learners, teachers, and design researchers, all of whom are not detached from phenomena in the design research. They are rather considered integral parts of them. Thus, epistemic processes are viewed as processes of coming-to-know insight with, from, and for others, rather than being independent of the people involved.
3. The axiological commitment pertains to what researchers value and how they determine that value, i.e., in our educational design research, design researchers are aware that their design choices prioritize certain aspects while potentially overlooking other equally relevant factors. Their goal is to comprehend how the design choices work in practice rather than attempting to control the research situation (e.g., through randomized controlled trials). Design researchers are particularly conscious that the effectiveness of their design choices depends on what the individuals involved prioritize in implementing the design (van der Wal et al., 2021), turning these choices into subjects for possible revision.

Design research generates a piece of reality through (re)design, and this piece of reality may vary depending on the (ontological) contexts in which it is intended to be implemented. As design researchers bear scientific responsibility for the designs they create, the implementation of these designs requires investigation to acquire knowledge in an (epistemological) process of theorizing about how the intended users interact with the design and contribute, or not, to achieving the goal. This, in turn, informs iterative steps in revising the design and, at times, even the goal itself. Hence, the epistemological commitment in design research appears ontologically grounded in shaping an educational 'way of being' – how it could or should manifest. In this process, each

design decision gives preference to certain aspects over others, thereby assigning value to these aspects while deferring other possibilities. Thus, design decisions are expressions of an axiological commitment.

Design research that is based on these commitments does not privilege research over practice; it is rather inspired by a stance that Akkerman, Bakker, and Penuel (2021) call ontological synchronizing meaning to take practice seriously and encounter the people involved on eye level, to attune the design with their lives and learn with, for and from them for theorizing. It even means to subordinate “one’s epistemological and axiological perspective to the ontology of people’s wider lives as they are and are always becoming anew” and to imagine “that publics will make sense of or act on the world in ways inspired or informed by our work, although from many different standpoints” (p. 422). Learning in this view is considered as “meaningful movement in motion” (p. 417) starting from a specific position guided by a purpose towards possibilities of students’ development.

These general considerations suggest that ontological, epistemological, and axiological commitments in our design research are related. However, they do not precisely articulate how they can be considered related and what these commitments have to do with the design and the local theory. To address this gap, this article is guided by the following research question: *How are the ontological, epistemological, and axiological commitments related in design research in education, and what does this relation have to do with the design and the local theory?* Answering this question is crucial as it has significant consequences both theoretically and practically. If design decisions exert a strong influence on the ontological and epistemological foundations of teaching and learning in practice, then those involved directly but mostly implicitly experience the pivotal role these foundations play in the teaching and learning processes through the design decisions. This will precisely affect the theorization of the empirical findings of design research.

To address this gap, in Chapter 2, I will elaborate in more detail on our design research of an online summer school (see Bikner-Ahsbabs et al., 2022). I will delve into the challenges posed by the shift in the 'way of being' we encountered during the pandemic when we decided to undertake this research. Furthermore, I will describe our process of coming to know the findings of this research. In Chapter 3, this design research example will serve as a foundation for scientific reflection that will unfold a frame concept for an ethico-onto-epistemological commitment as a unified commitment underlying the philosophical position of our study, and consider the general applicability of this concept. Due to ethical considerations, information about students and their PhD projects will be anonymized or left out.

2.0 Designing an online summer school at the beginning of the pandemic in 2020

Due to the onset of the Covid-19 pandemic in spring 2020, the 10th summer school of the Young European Society for Research in Mathematics Education, YESS10, planned for a whole week, was canceled. Two young researchers, Andrea Maffia and Dorota Lembrér, and two experts of the summer school, Jana Trgalová and Angelika Bikner-Ahsbabs, wanted to switch to an online conference system to run the summer school. Given the lack of knowledge about distance learning for early career researchers and the sudden lockdown situation, the team launched a design research study with only three weeks of preparation time.

In this design study, Angelika Bikner-Ahsbabs and Jana Trgalová guided the two thematic working groups (TWGs) to which they were invited. Bikner-Ahsbabs chaired TWG7, focusing on theories, language, and representations, while Trgalová chaired TWG5 centered on technology. Andrea Maffia and Dorota Lembrér, participants in previous YESS conferences and still belonging to the group of young researchers, functioned as brokers (Akkerman & Bakker, 2011), who bridged various levels of expertise, handled the technical background of the virtual spaces, and used social media and digital tools to assist both the students and us in conducting the summer school. Additionally, Arthur Bakker joined the research team as an external interviewer.

The team tackled the challenge of designing the two TWGs within three weeks by leveraging design decisions developed iteratively in previous YESS conferences. These decisions were adapted to address the lockdown situation of the pandemic, taking into account diverse home situations of students. Transitioning from the physical spaces of previous summer schools to a virtual environment, we acknowledged that the space was not merely given; rather, it had to be regarded as an ontological condition to be theorized and designed.

2.1 Theorizing the virtual space

Lefebvre (1991) regards the space as a social construction continually produced and reproduced by the people who utilize it. He conceptualizes the production of space through three dimensions: spatial practice, illustrating how the space is used; conceived space, representing how the space is planned and constructed; and lived space, referring to the representational aspect of space as it is experienced through social creation, spatial practices, symbols and images.

In the virtual space of the conference system we used, the technological-material architecture determines how the space is conceived and what can or cannot be done. Regarding individual students' home situation, the computer is located within another space – their material home space – further determining what can and cannot be done with the computer for participation in the virtual space. For instance, if two people share the same room at home, arrangements must be made for participating in the virtual summer school.

In the virtual environment, layouts provided by the conference system were designed to accommodate spatial practices. Participants could write and store minutes, exchange comments in a chat room, convene

in breakout rooms for small group discussions, share drawings on a whiteboard, and utilize options to search for references outside the system that could be imported for distribution within the system. All these tools were represented by symbols, which the students needed to learn to interpret and use. Similarly, students had to familiarize themselves with other symbolically represented spaces, such as meeting spaces for the lecture room and breakout groups. The two brokers produced a manual to assist participants in navigating the conference system and organized a pre-conference meeting to ensure that each student could access their virtual space, thereby introducing them to their new environment.

The social production of space was also influenced by our design choices, shaping the lived space that could be researched by interrogating the students' experiences. The spatial circumstances of the students prompted us to consider the importance of boundary crossing (Akkerman & Bakker, 2011) between different social practices in various virtual spaces and digital tools that students might use, as well as between our virtual space and their (home) spaces. The effectiveness of such boundary crossing and the utilization of our virtual space heavily relied on the individual internet connection at home.

In addition to boundary crossing between their home space and the virtual summer school space, students also had to establish personal boundaries, such as putting their smartphones aside or closing email accounts in their home situations to effectively manage their daily participation at YESS10. Whenever possible, the brokers assisted the students in managing their private situations related to the summer school, ensuring they returned to the virtual space in case of technical exclusion. To facilitate this, chat groups were established to exchange information quickly.

2.2 Structuring and designing

Three design choices, aligned with three design principles (Table 1), guided the design of the virtual space in the conference system in the TWGs. Design Choice 1 involves structuring the learning of research through a diagram, while Design Choices 2 and 3 concentrate on the practical application of the diagram.

Table 1: Design principles relate to the design choices

Design	Design principle
Choice 1	Sharing a diagrammatic structure (research pentagon, Figure 1) that keeps invariant.
Choice 2	Setting up critical friends' feedback as a starting point for engaging the students in social interactions and in-depth discussions.
Choice 3	Organizing time and space by clearly interwoven linear and cyclic rhythms of the diagram use.

2.2.1 Design Choice 1

The primary objective of this choice was to establish a foundational structure for what students were expected to learn during the summer school – specifically to gain insights into research by advancing their own PhD projects. This goal was achieved by using the research pentagon (Figure 1), a diagram that integrates the five key components of conducting research (Bikner-Ahsbabs, 2019), to empower students to structure their individual research projects. The diagram structure remained invariant across all sessions, even as the specific content of students' research projects varied. It encompasses five crucial research aspects (Bikner-Ahsbabs & Prediger, 2010): research object, research aim, research question, research method, and research situation. The research situation is further divided into two levels: the internal and external situations. The external situation involves the discourse within the field to inform constructing a theoretical framework, while the internal research situation pertains to the circumstances of conducting the research method, including those to be considered during the implementation of the design. According to Variation Theory, the Design Choice 1 enhances the possibility of learning how to conduct research by discerning the invariant research aspects indicated by the pentagon in both their projects and those of their peers (Orgill, 2012).

The research pentagon represents not only the research aspects by the vertices but also all the connections between them to be considered during research. For example, the research aim should arise from a problem to ensure the relevance of the research. Answering the research question should contribute to solving the problem, thereby achieving the aim. The research question should revolve around the research object and inform the research method that aids in shaping the (internal) research situation. The theoretical framework, established based on knowledge in the field (external situation), should provide an opportunity to conceptualize the research object.

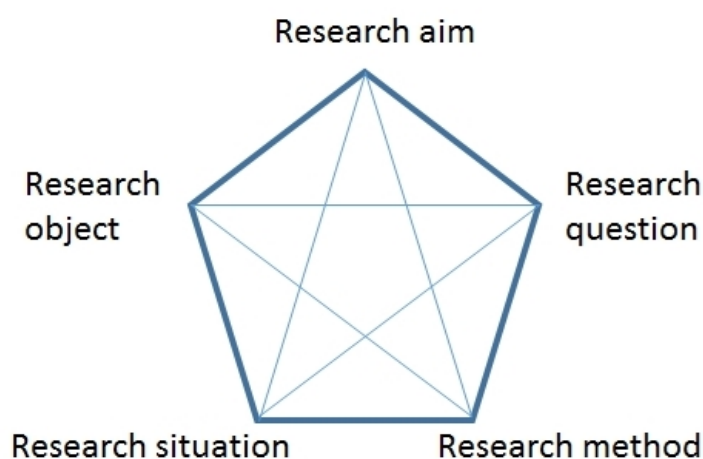


Figure 1: The research pentagon (Bikner-Ahsbabs et al., 2022, p. 2939, Figure 1; Bikner-Ahsbabs, 2019, p. 154)

The research pentagon emphasizes research aspects relevant for our learning goal, but this tool does not inherently specify how it should be used. The following Design Choices 2 and 3 framed how the research pentagon was applied in the sessions of the school.

2.2.2 Design Choice 2

The objective of the Design Choice 2 was to engage the students and enhance discussions (Table 1). Before the summer school, we established pairs of critical friends in each TWG. In addition to reading the submitted papers on the students' projects in the TWG, their task was to delve more intensively into the critical friend's project to provide informed feedback in a presentation. We anticipated that the students would use the research pentagon to structure feedback and that Choice 2 would enhance collaboration within the group.

2.2.3 Design Choice 3

The aim of Choice 3 was to address the new ontological conditions of meeting via a conference system but avoid increasing the complexity in the virtual space. We decided to structure time through establishing a clear rhythm for the organization of the TWGs. This choice proved to be more significant than anticipated initially.

According to Lefebvre (2004), we can define rhythm as variation in repetition, linking time, space, and energy, and distinguish between linear and cyclic rhythm. The ten sessions (S1-S10) we prepared followed both types of rhythms (Figure 2). Linear rhythm adhered to the regular clock time when the sessions occurred. In a cyclic rhythm, each session followed a procedure involving six ways of using the pentagon, albeit with changes and variations. Pairs of critical friends initiated each session, but the individuals and their topics changed, and this also varied based on the ways presentations were conducted. Moreover, the pentagon usages c, d, e (Figure 2) also underwent variations. The timing and the manner in which breakout groups took place were adjusted based on the students' needs.

These design choices were not arbitrary, they were rather motivated by the specificity of the situation. In the lockdown situations, nearly all types of rhythms that usually determine everyday life had disappeared. Our choice for a clear rhythmic structure in time and space was driven by the anticipated need of students to better coordinate their participation with their home situation. The other design choices had been developed over the past decade in previous YESS conferences, where they demonstrated their effectiveness in fostering learning to conduct research through feedback and discussions from, for and with each other across various topics. All students and the experts were expected to use the research pentagon on the shared screen to communicate about research. Therefore, the research pentagon had the potential to function as a boundary object (Akkerman & Bakker, 2011), bridging the social practices of the research cultures the participants brought in.

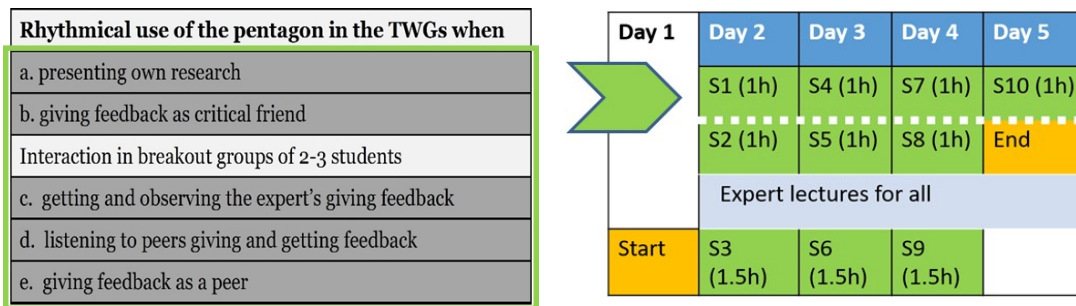


Figure 2: Rhythmic organization of the summer school sessions (Bikner-Ahsbals et al., 2022, p. 2939, Figure 1)

2.3 Theorizing the use of the research pentagon (retrospectively)

For the students, the research pentagon was, first and foremost, an artifact – that is, an object created by human beings. The instrumental approach (Vérillon & Rabardel, 1995; Rabardel, 2002) helped us understand how a learner could effectively appropriate the artifact. In this approach, the artifact is distinguished from an instrument, which is a hybrid combination of the artifact and schemes of use for a specific class of situations (Trouche, 2020a; 2020b). The instrumental genesis of this artifact is the process by which a student transforms the artifact into an instrument, characterized by two dialectical sub-processes: instrumentation and instrumentalization. Instrumentation is the process through which the artifact and its possibilities shape the way a person can deal with it. Instrumentalization works in the opposite direction; it is the process by which the students personalize the artifact based on their knowledge or for their own purposes and needs.

In then two TWGs, the students underwent a process of instrumental genesis, transforming the pentagon into an instrument. The experts in the TWGs orchestrated these instrumental geneses, considering the three components of instrumental orchestration (Trouche, 2004):

1. Didactical configuration: This involves “a configuration of the teaching setting and the artefacts involved in it” (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010, p. 215).
2. Exploitation mode: This is “the way the teacher decides to exploit a didactical configuration for the benefit of his or her didactical intentions” (p. 215).
3. Didactical performance: This encompasses “the ad hoc decisions taken while teaching on how to actually perform in the chosen didactic configuration and exploitation mode” (p. 215).

In our design research study, we structured the instrumental orchestrations in a rhythmic manner and explored this specification of instrumental orchestration by addressing the following research question: *How did the students experience the rhythmic orchestration of pentagon use and the pentagon itself as contributing to their learning about research?*

2.4 Methodical approach of the design experiment

The introduction of the research pentagon varied between the two TWGs. In TWG7, students were required to read an article about it before the summer school (see Bikner-Ahsbabs et al., 2022), while in TWG5, it was introduced in the first session. This difference provided an opportunity to explore whether learning to use the research pentagon could be conceptualized as instrumental genesis.

	Stud. 1	Stud. 2	Stud. 3	Stud. 4	Stud. 5	Stud. 6	Stud. 7	Stud. 8	Stud. 9	Stud. 10	Stud. 11	Summ.
a. Presentation	→	→	→	→	→	→	→	→	→	→	→	→
b. CF-feedback	→	→	→	→	→	→	→	→	→	→	→	→
Breakout interaction	→	→	→	→	→	→	→	→	→	→	→	→
Discussion with	→	→	→	→	→	→	→	→	→	→	→	→
c. Expert feedback	→	→	→	→	→	→	→	→	→	→	→	→
d. Listening to peers	→	→	→	→	→	→	→	→	→	→	→	→
e. Peer feedback	→	→	→	→	→	→	→	→	→	→	→	→
Summary	→	→	→	→	→	→	→	→	→	→	→	→

Note: Green arrows: 1st vertical and horizontal analysis. Blue vertical arrows: 2nd vertical analysis

Figure 3: Analyses procedure (Bikner-Ahsbabs et al., 2022, p. 2942, Figure 2)

Eleven volunteers out of the 20 participants agreed to be interviewed after the school. The interviews were conducted by Arthur Bakker, an expert on boundary crossing, who was not directly involved in the summer school. Given that boundary crossing was considered the main challenge expected to influence the pentagon use, the rhythmic organization, and the teaching and learning in the summer school, the interview scheme was structured to address this aspect, i.e., to:

1. Elicit boundaries, by asking about challenges.
2. Elicit insight into boundary crossing, by asking about efforts put into these challenges to identify learning mechanisms.
3. Elicit how the pentagon functioned as a boundary object to explore the role of rhythm.
4. Identify the role of brokers.
5. Elicit how learning about research took place.

First, a general reflection with all the eleven students and then individual interviews were videotaped. All video data were transcribed verbatim using pseudonyms for the students' names. In addition to that, the brokers and the experts took notes during the summer school.

The complete team of five design researchers analyzed the data in three steps and monitored the analyses by regular digital meetings (Figure 3):

- In Step 1, the research team conducted a vertical analysis of the individual interviews guided by the concepts of boundary crossing and instrumental genesis of the pentagon. For each interview, three researchers were involved in the analysis one

by one. In the first round, one researcher coded the interview data deductively and inductively (Mayring, 2015). The second researcher validated the codes and added comments. The third researcher further validated the codes and comments and summarized the results. Disagreements were discussed within the research team, leading to a final decision. Based on the summaries, the two brokers undertook an individual rhythm analysis: They identified transcript excerpts related to the five kinds of pentagon use and the codes on learning and inserted them into the matrix (Figure 3) with a final summary each (light green vertical arrows).

- In Step 2, the research team conducted a horizontal analysis of the collective experience twice. First, a collective rhythm analysis was performed by comparing and contrasting the students' experience based on the five kinds of pentagon use. Second, by comparing and contrasting the individual summaries, we identified patterns related to instrumental genesis across individuals (dark green horizontal arrows).
- In Step 3, the team conducted another vertical analysis of the students' reflections on learning about research and identified instances of this kind of learning related to the pentagon use (blue arrows).

2.5 Findings

In this section, I will primarily focus on the research pentagon as a boundary object, its instrumental geneses and orchestration, and the students' reflections on learning.

The use of the research pentagon was not a straightforward process. The students of TWG7 had read the article on the research pentagon before the summer school and used it directly for presenting their research and providing feedback. Thus, the processes of instrumental genesis of the research pentagon had already started before the summer school. This was different in TWG5. There the process of instrumental genesis started in the first sessions. This became evident in the way the first two presenters expressed their uncertainty related to the expert's use of the pentagon in the first session. For example, Nordy described his first encounter with the pentagon as a challenge for which he needed time:

I really think that it could be a great thing to organize your work and to give also qualitative, highly qualitative feedback to others but as previously mentioned in our general feedback talk, I need some more time to read the whole text and to incorporate it, to think about this research pentagon (Nordy)

This was a clear indication that schemes of use were not so easy to establish. However, the process of instrument development had begun, but in the end, Nordy was still not satisfied:

... our understanding of the research pentagon, we tried our best, and it was ok, but the feedback referring to the research pentagon had, could have been deeper if we get a better understanding of it. (Nurdy)

In both TWGs, we have identified instrumental geneses in our analyses. I will now summarize and illustrate them with some excerpts and then move on to the topic of learning, including the role of rhythm.

Findings from the analysis in Step 1:

Findings pertaining to individual instrumental genesis indicate that the pentagon evolved into a personal instrument for:

- scrutinizing one's own research and ensuring its coherence
- analyzing the research projects of others
- structuring feedback by the critical friends

Emma initiated her engagement with instrumental genesis prior to the summer school, articulating her utilization of the pentagon as a means to scrutinize her own research and ensure its coherence.

I was actually visiting Angelika in October and we worked on the pentagon then I was quite familiar with it, but I still think it's very good to come back to it now and then. Also, just to see that you are making progress or making your project more coherent because I think it's very useful to talk about; if there's some parts of your research project that is not coherent (Emma)

Emma's description further reveals her application of the pentagon as a tool for analyzing the research projects of others:

I used the pentagon when I read the papers, for instance when I read through my critical friend's paper and whether I found some indications of the research aim and the research object, research questions, the method and... (Emma)

The case of Kira serves as an illustration of using an instrument to organize feedback, as evident in her statement:

After the first day of presentation in the group, when Angelika gave the talk on the research pentagon. It was on Tuesday I think, I read the paper again and restructured also my feedback slides according to the research pentagon. And it kind of helped me to get more structure into the paper of Victor. (Kira)

Addressing her specific needs, Kira also provided an example of instrumentalization:

... I will take the research pentagon into a small group discussion at the university, so we have this weekly meeting with the young career researchers from our department and I will present it. (Kira)

Findings from the analysis in Step 2:

Throughout the individual interviews, a recurring theme emerged – listening played a pivotal role in the students' learning. This emphasis on active listening was reinforced by the cyclic rhythm, as highlighted by Peter. He underscored the significance of being able to repeatedly listen to and observe the use of the pentagon by others: “I had the ability to see the pentagon in action for ten times.” Dan explained that listening was crucial in this repeated use: “Doing it like a few times to other colleagues, each one in a different subject area, and hearing the other teammates, how they raise questions, was really really valuable. I mean, I learnt from it a lot.”

It was the cyclic rhythm, characterized by the repetition of the same in varying ways, which enabled the students to observe the use of the pentagon. When seeing and listening to others how they did “it like a few times to other colleagues”, Dan and also other students expressed what nurtured them to learn. For example, Peter found that the effort he invested was valuable by “following other’s line of thought” and Kira pointed to the value to learn from others to “think out of the box”, for example by listening to: “Myrka, she focused creativity and a special topic which was for me like *thinking outside the box* because we always research very topic specific and not taking into account something broader, maybe.”

Findings from the analysis in Step 3:

Findings coming from reflecting on learning that took place addressed the *What* and the *How* of learning.

Concerning the *What*, the students learned about the existence of “blind spots” in their own research, represented by missing “vertices”. This awareness led to a gradual understanding of the “edges” as representing connections, ultimately contributing to an enhancement in the coherence of their research. For example, Kira said “then I could more clearly identify what was missing for me and what was also good.” She further stated:

When I thought about Victor’s paper [the critical friend] and he, for example, [...] filling the link to the research question would mean to ask myself, or for him to ask himself the question ‘How can I measure this?’. What method can I use in order to find these competencies, to identify these competencies. [...] Is that the method that’s correctly measuring these competencies? (Kira)

In terms of the *How*, the primary mode of learning for the students involved listening to their peers and how they used the pentagon. As outlined in Step 2, this learning process occurred repeatedly, guided by the cyclic rhythm. Surgeryfish for instance reflected on her learning: “what I see from others [when pointing to the vertices] and fit into my work [as a possibility to be used]. ... A kind of learning, becoming aware [of the aspects in the pentagon related to own research].” Myrka also observed [and listened to] how others utilized the pentagon in various ways, showcasing possibilities for her own approach. Learning from

her peers, she discovered that observing others not only facilitated communication about theories but also made it possible to transcend the boundaries of theoretical cultures:

... but once you see how other students used it, I think it's a very nice way to use it [the pentagon] your own way, personally, everyone used it; they didn't use it the same way. Some of them approached it in a more specific [way] and they could talk about theories more. (Kira)

The students spoke of *seeing* the use of the pentagon on the shared screen of the virtual space by pointing with a pointer or the cursor. However, *seeing* did never happen without *talking* about the pentagon use. What the students meant is seeing where others pointed in a conversation about research. Hence, seeing entails listening. Listening can take different forms depending on the focus or purpose of listening. We identified three kinds of listening:

- listening to understand research of others,
- listening to explore how to assist others in their research and
- listening for the purpose to improve one's own project.

Myrka expressed how she listened to understand her peers' research:

It is a very nice tool that helps me organize the aspects of everyone's research because its aspect [related to the vertices] has... it's dynamic and I think it's very important to keep the right focus on each one of them [the vertices]. (Myrka)

Here she learned that the five aspects are to be used dynamically. Red listened to explore how to assist others in their research:

Usually we responded to questions posed by the presenters. Like the core questions that they needed help with. So we took turns trying to figure out how to help them. (Red)

Finally, Dan listened to improve his own project:

It's good to have teammates, you know, looking to what you're doing and maybe lighting some points in areas which you just took in without notice. (Dan)

Dan described his learning process as an awakening to aspects within the pentagon that had gone unnoticed before. It's noteworthy that the three distinct intentions to listen, in a sense, align with the development of the three instruments the students created. Consequently, instrumental genesis and listening seemed to be closely related in the students' learning processes.

2.6 Theorizing summary

The research pentagon found application in all presentations, excluding the initial ones in TWG5. Both students and experts consistently

and increasingly made references to it in their various feedback comments. Its utilization on a shared screen created collaborative opportunities through deixis, with a pointer or cursor in the virtual space enabling the audience to “see” and “hear” where the students pointed and what they articulated. Our sensory-motor systems were engaged, encompassing activities like seeing, hearing, listening, talking, pointing, and acting collectively on a shared object. This dynamic turned the pentagon into a shared reference space for *object-related social interaction*, fostering joint attention (Balantani & Lázaro, 2021; Diessel, 2006; Knoblich, Butterfill, & Sebanz, 2011; Sebanz, Bekkering, & Knoblich, 2006; Shvarts, 2018; Stukenbrock, 2015), which can be regarded as a theoretical precision of the nature of learning happening in the settings.

Joint attention played a crucial role in coordinating attention with co-participants in communication. Throughout this process, rhythm emerged as a key element, supporting the instrumental genesis of the pentagon. This led to observable patterns or schemes of pentagon use on the shared screen during the TWGs, facilitated by the cyclic rhythm inherent in the design. The repetition of pentagon use – employing the same research aspects in varying ways – allowed students to witness the growth and expansion of both their own knowledge and that of their peers. For instance, Myrka detailed how others learned to be more specific: “I think there were some students who couldn’t find... who couldn’t use it [the pentagon] in a specific way, but during the discussions [in using the pentagon] of their feedback, they realized that they have to be more specific.”

Thus, the students experienced growing knowledge related to themselves and the group where active listening played a crucial role.

This led to the conceptualization of rhythmic instrumental orchestration, which combines two theoretical perspectives through specification (Bikner-Ahsbals et al., 2022). Specifically, this orchestration involves merging linear rhythm in the didactical configuration of sessions and cyclic rhythm in the exploitation mode for addressing and discussing research, rooted in individual instrumental geneses. The ways in which the expert employed the pentagon in her didactical performance were highly relevant, where the pentagon served as a reference space that directly supported joint attention in the TWGs as a fundamental condition for mutual learning.

In conclusion, in the implementation the three design principles worked together, complementing and reinforcing each other throughout the entire summer school. The establishment of a shared reference space for joint attention was realized through the common but varying use of the pentagon diagram as a boundary object, fostering both individual and collective learning. However, the utilization of the research pentagon was not straightforward; the pentagon needed to be instrumented, collectively supported by the rhythmic orchestration. In each session, a pair of critical friends initiated this process, creating a recurring experience throughout the school that contributed to the development of expertise in the group, where the importance of observing and listening for learning became evident. Summing up, the synergetic connection among the three design principles suggests

promising possibilities for designing teaching and learning experiences, extending beyond the scope of summer schools.

3.0 Reflection on the relational nature of epistemology and ontology

This design case elucidates the fundamental commitments underlying design decisions in educational design research. The necessity for design research arose due to a sudden ontological shift in society, significantly impacting the education of international PhD students in mathematics education. The decision to embark on a design study, even under time constraints, reflects the specific axiological character of design decision, offering insight into the ethical commitment of the researchers' professional responsibility (Boylan, 2016) to the students. Despite global lockdowns, we determined that the summer school was crucial and should proceed in some form. Opting for an ontological shift, we chose to conduct the summer school via a conference system as it provided a straightforward and accessible means for all participants to present, discuss, and exchange their research projects, similar to a face-to-face meeting. Initially, we possessed a naïve ontological understanding, intending to preserve learning in groups. We believed that maintaining this collaborative learning experience would be beneficial for the students, allowing them to derive insight from being and learning together.

- How can we understand the notion of relational ontology more precisely?

When the students entered the school, the multiplicity of others' presence opened up possibilities for feedback, comments, and advice to each student (see Benjamin, 2015). During the preparation of their presentations, this multiplicity was to be considered initially as a potential and then enacted and experienced when using the research pentagon as a common reference space. Thus, the multiplicity of scientific backgrounds of the students and the expert and the singularity of each individual created a dialectic that determines relational ontology, expressed through *being-in-relation* (p. 8). Our research results confirm the students' acting by being-in-relation, highlighting the significance of using the pentagon as a diagrammatically shared reference space within a rhythmic orchestration that facilitates access for everyone through joint attention. In particular, the Design Choice 2, involving the establishment of pairs of critical friends, served as a means to initiate being-in-relation. This was observable in each session as the starting point for in-depth discussions.

This ontological commitment aligns seamlessly with our epistemological stance, wherein students acquired knowledge, termed *coming-to-know*, through a process of giving and receiving feedback with, from, and for others in the class, including the experts. According to our design choices, students presented their projects to their peers and received feedback, specifically from their critical friends. Advice came from both experts and peers, all guided by the research pentagon. This dynamic unfolded for all students in the course, each navigating a dis-

tinct stage in their individual development. Consequently, the evolution of each student's learning process, as a process of coming-to-know insight, was intricately tied to the interactions with other students and the responses of the expert. Thus, the group's epistemology took on a relational nature, emphasizing that the process of coming-to-know insight was firmly rooted in the earlier outlined relational ontology.

- How can we understand relational epistemology more precisely?

The concept of relational epistemology is about the students' ways of coming-to-know insight based on the task, the tools provided, their peers, the teacher and the brokers. Coming to know insight happened with, from and for each other, for example by observing of and listening to how the pentagon was used in the groups. This relational epistemology of the groups led to a collective experience of coming-to-know insight, which enabled the participating students to inform the interviewer in the data collection about the learning related to themselves and the group.

A similar but also contrasting concept has been introduced by Abtahi (2022). She coined the concept of relationship epistemology in her research on mathematics education with indigenous people and stressed related to this epistemology the ethical responsibility she as a teacher felt. In this perspective, knowing may come from everywhere depending on various kinds of interrelated relationships we are living in, relationships with humans and non-humans in the social, cultural, linguistic and natural world.

This epistemology is not about an individual being in possession of intellectual ability or of knowledge or understanding about something. Instead, it is about knowing one's position in interpreting and engaging with the webs of other things (such as mathematical concepts) and with the multiplicities of one's experiences. In this epistemology, the goal is to capture the idea that knowledges are mutually dependent and are interrelated to ever-evolving life and cultural experiences (p. 156)

In contrast, our concept of relational epistemology focuses on the interconnectedness of transformative processes among students within a specific arrangement crafted for learning with, from, and for others. This is exemplified by our most noteworthy result – the learning through listening – an experience shared by the entire group. Notably, in a summer school setting, the act of listening is only feasible in connection with, from, and for others, thus firmly rooted in a relational ontology.

- How can we understand the way relational ontology and epistemology hang together?

Relational ontology and relational epistemology were not pre-existing in our study; instead, they were actively brought into play as interconnected phenomena during the TWGs. This interconnection material-

ized in the becoming-with (Hoppe & Lemke, 2021, p. 129) of each student who engaged in a common process of coming-to-know insight. This process unfolded as a collective and individual experience of growing knowledge for each student.

Initially, the rhythmic organization of the sessions in a conference system aimed to assist students in navigating and setting boundaries in the lockdown situation. Surprisingly, this rhythm not only served its intended purpose but also proved to be beneficial for learning about research, a result we could underpin using rhythm analyses (Lefebvre, 2004) and the concept of rhythmic instrumental orchestration. A crucial aspect of the rhythmic organization was the cyclic rhythm within each session. While all students collectively experienced the rhythmic structure of pentagon use, each individual experienced it time-dependently, based on their state of becoming-with, influenced by their exchange of ideas with, from, and for others. The research pentagon served as a shared reference space for joint attention, but each student's experience was unique, contingent on their distinct ways of learning. Consequently, the students' being-in-relation and their process of coming-to-know insight can be regarded as intertwined, influencing their becoming-with and contributing to the growth of research knowledge within the group.

3.1 Reflection on ethics

In the realm of mathematics education, there are nuanced explorations of ethical approaches that extend beyond conventional concepts of ethics in general teaching (Sabbagh, 2009). Notable examples include ethics as the fundamental philosophy of mathematics education (Ernest, 2012), ethics as the call for reflexivity in practice (Stinson, 2017), and ethics as a navigation between being-with-others, societal and cultural aspects, ecological considerations, and the self (Boylan, 2016). When considering theories on teaching and learning, a more profound conceptualization is required, one that acknowledges the inherently social nature of these processes (Ernest, 2019), addresses responsibility and care (Ernest, 2019), and upholds a commitment to the content of mathematics education research, as seen in this study, or mathematics in the context of teaching the subject. An intriguing candidate for such a concept is the recently introduced notion of communitarian ethics by Radford (2022). Ethical action revolves around discerning right and wrong or good and bad behavior, which may be grounded in values embedded in design decisions. Consequently, ethics becomes an integral part of axiological commitments. Radford interprets ethics as forms of alterity concerning the self and the other, shaped by responsibility, commitments (to learning), and care.

To clarify whether this kind of ethics has been practiced in our design study we have to look back to the way the virtual space is conceptualized and used. The reason is that decisions about how space is used can be fundamentally ethical. For instance, a student providing care in the home space during lockdown could face an ethical dilemma if an expert asks him or her to take responsibility for writing minutes and therefore also caring for their peers.

Responsibility means “living and acting with and for others; ... to respond to the call of others as they are on their own terms” (being-for-others) (para 6.5.1). With respect to our research team, responsibility was practiced when the virtual space was designed. A manual written for the students, checked by the experts and presented in a pre-conference meeting ensured that the virtual space was accessible for all the students in the two TWGs. In addition, the brokers were present to respond to the students’ needs related to the summer school. It is remarkable that in the course of the school, the students repeatedly took freely responsibility over for others, e.g., through writing and storing minutes, sharing information and providing and searching for references.

Care “is to go beyond ourselves and to be dragged powerfully into the world, to position ourselves there, with-the-other” (para 5.2.3). The two brokers ensured care for the students by being present to provide assistance (e.g., to get students back into the system in case the system excluded them), store ideas and provide information for those who could not attend the meeting because of family affairs or other reasons, and to provide references immediately if necessary. The brokers had also established chat groups via social media to provide a channel for unpredictable information in both directions, e.g., if a student could not participate.

Commitment “is both the promise and its realization of doing everything possible to work side by side with others in the course of our joint labour (e.g., trying to understand the process being followed to solve a problem, trying to contribute to the classroom common work)” (para 6.5.2). Such commitment already began with establishing pairs of critical friends who mutually tried to understand the projects and the problems of the critical friend and could translate them to the group and the expert in their feedback presentation. As a side effect, these pairs of critical friends exchanged information even before the summer school and thereby strengthened their mutual commitments. The use of the pentagon supported the participants’ commitments to research by the five ways of pentagon use that ensured that various kinds of feedback were possible for each student’s presentation.

3.2 Fusion of the ontological, epistemological and ethical commitments

As I will elaborate, the three commitments are not independent; on the contrary, they are deeply intertwined and could be considered as the three dimensions of one unified commitment. Leaning on Barad (2012), I argue for a fusion of the ontological and epistemological commitments with ethics as the three dimensions of an *ethico-onto-epistemological commitment* for design research. A potential advantage of such a unified frame concept would be that design decisions based on the three commitments could be reconciled in the course of the design process incorporating the design researchers’ responsibility of the two final products conceptually.



Figure 4: The transforming role of a unified ethico-onto-epistemological commitment

As extracted from our design research study, our ontological commitment is relational considering the way of being-in-relation (relational ontology) of the participants in the group. It manifests hand in hand with the participants' relational ways of coming-to-know insight (relational epistemology). The two commitments are intertwined because the way of coming-to-know insight is ontologically grounded but also renews ways of being-in-relation, while both foster the students' becoming-with in processes of growing knowledge. Given we would strengthen the students' individual learning while ignoring the others in the group, this would not only affect the relationality of the epistemological commitment but also downgrade the relational nature of ontology in the TWG. Becoming-with in the group would be hardly possible. This would also affect the ethical commitment. Even if the two brokers would still feel responsible for and care about the students, the commitment to the learning content within the group would most likely disappear as this relates to the relational way of students' learning about research from, with and for others. Therefore, the ontological, epistemological and ethical commitments are not independent, they are intertwined in our design research case and should thus be regarded as fused into one, which nurtures *becoming-with in a community of growing knowledge*. In this case, communitarian ethics shapes a transformative atmosphere in which becoming-with in processes of growing knowledge is possibly based on the intertwined practices rooted in relational ontological and relational epistemological assumptions (Figure 4).

Similarly to Barad's (2012) elaboration, phenomena being evoked by design create reality. If a design is developed with responsibility and care, the design is expected to produce certain key phenomena when adapted appropriately to a new context, e.g., listening as a key factor of learning. The fusion of the commitments into one relational ethico-onto-epistemological commitment reminds design researchers to be also aware of their responsibility for the design even when used "in ways inspired or informed by our work, although from many different standpoints" (Akkerman et al., 2021, p. 422). In this case, design researchers are still responsible, meaning that they should take care that applicants have the chance to understand and use their design in the intended manner while applicants are free to do so or make it their

own design for a different purpose. This requires from design researchers to be transparent about the local theory as well as the underlying ethico-onto-epistemological assumptions.

3.3 Design and local theory based on a unified ethico-onto-epistemological commitment

How does the idea of the fusion of commitments help us understand the interrelatedness of design and local theory in a design research study? Let us reconsider our design research example. It was built upon previous design experiences, which influenced the current design choices. Additionally, what emerged was the relevance of rhythm, which, I assume, was implicitly present in some way before. In a face-to-face situation, such as a summer school where a teacher has room for improvisation, the rhythm of the course may not necessarily need to be explicitly addressed. Due to the responsibility and care required when teaching via a conference system, rhythm was placed at the center as a crucial design choice, acting as a counterpoint to the consequences of the lockdown. However, the design decision on rhythm did not only have an ontological effect; it also had an effect on the epistemological level. Unexpectedly, our theorizing highlighted the relevance of rhythm also for learning, specifically for learning through listening. This was uncovered by a systematic rhythm analysis of the use of the research pentagon, serving as a common reference space for joint attention among the students. Thus, the new design decision on rhythm had simultaneously both an ontological and an epistemological effect, both entangled in the design process, seamlessly embedded in the ethical framing of conducting the summer school.

Considering one unified ethico-onto-epistemological commitment, the design in our example expresses a crystallized form of the 'way-of-being', reflecting our relational ontological commitment while keeping the 'way of coming-to-know' insight implicit. Conversely, the local theory reflects the result of our scientific 'way of coming-to-know' insight, in the research team with the students, resonating with our relational epistemological commitments while keeping the 'way-of-being' implicit. Design and local theory were entangled through the cyclic development process, in which we, as teachers, brokers, and design researchers, have practiced communitarian ethics, emphasizing ethical responsibility for both, the design and the local theory.

4.0 Conclusion

The elaboration of the fusion of ontological, epistemological, and ethical commitments primarily stems from a single design research study. Can this research case be regarded as an example of a more general phenomenon? I think yes, if we conceive ethical-onto-epistemological commitment as a three-dimensional space with the three commitments as its dimensions. But like any research endeavor, educational

design research is typically conducted within the framework of a background theory, such as social constructivism or activity theory. This background theory delineates the scope of what is considered a suitable research object, research aim, research situation, or answerable research question (Mason & Waywood, 1996; Prediger et al., 2015). The coherence of such a background theory relies on certain assumptions, of which ontological, epistemological, and axiological assumptions are the basic ones, specific to each theory, and relevant for the conceptualization of design research (see Bakker, 2018). Therefore, the fusion of the three commitments should also be specific for each background theory.

The exemplary case of design research discussed above is rooted in an object-based form of social constructivism, wherein the object adopts the semiotic form of a diagram of the research pentagon, and the social relations of the participants are central to the design of teaching and learning. Related to this case, I have elaborated on an interpretation of this fusion, but it is not clear how the three philosophical commitments interrelate in other instances of design research, conceivably forming a three-dimensional space. In this respect, further research is needed. Referring to the design research example, it seems plausible to transfer insights to future design research within similar background theories, particularly in terms of fostering transparent communication about reconciling underlying commitments and understanding their potential impact on teaching practices. Therefore, it is crucial to approach such transfers with sensitivity, adapting insights to new conditions and contexts responsibly and with care.

Acknowledgement

I thank Arthur Bakker, Dorota Lembrér, Andrea Maffia and Jana Trgalová for their engaging contributions to the YESS10-design study and for their agreement to include our YESS10-study as an example into this article. Without their work, the design research study would not have been possible. I further thank Viviane Durand-Guerrier for taking up of the responsibility to organize this summer school online with an accompanying lecture program. (Ethical approval of the design research study by the Science-Geo Ethics Review Board of Utrecht University, No. Bèta S-20390)

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Journal Details EDeR – Educational Design Research
An International Journal for Design-Based Research in Education
ISSN: 2511-0667
uhh.de/EDeR
#EDeRJournal (our hashtag on social media services)

Published by

Hamburg Center for University Teaching and Learning (HUL)
University of Hamburg
Schlüterstraße 51
20146 Hamburg
Germany
+49 40 42838-9640
+49 40 42838-9650 (fax)
EDeR.HUL@uni-hamburg.de

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In collaboration with

Hamburg University Press

Verlag der Staats- und Universitätsbibliothek Hamburg –

Landesbetrieb

Von-Melle-Park 3

20146 Hamburg

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+49 40 42838 7146

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