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Title **Primary students' participation in mathematical reasoning: Coordinating reciprocal teaching and systemic functional linguistics to support reasoning in the Swedish context**

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Abstract The practice of reasoning has been regarded as core element for developing mathematical arguments. However, curricula reforms have only recently focused on reasoning as an essential part of mathematics education. In Sweden, the emphasis on systematically developing reasoning competences began in 1994 and became even more explicitly focused in the 2011 mathematics curricula reform documents. However, many studies, including our own, show that students and teachers face difficulties in conceiving what reasoning might mean and also how its growth can be supported in the everyday mathematics classroom setting. Through a collaborative intervention study, the present paper explores how the coordination amongst the basic tenets of Reciprocal Teaching and Systemic Functional Linguistics perspectives could potentially create a pedagogic design for Grade 4 students' reasoning in specific mathematical tasks.

Keywords collaborative design study
explaining
reading and writing
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Primary students' participation in mathematical reasoning: Coordinating reciprocal teaching and systemic functional linguistics to support reasoning in the Swedish context

Cecilia Segerby | Anna Chronaki

1.0 Introduction

In several curricula, reasoning has been pointed out as one of the basic competences in the learning of school mathematics (see, for example, Kilpatrick, 2001; Lithner, 2006; Niss, 2003) but also for societal participation (Chronaki, 2018; Hanna, 2000). And it is discussed in direct relation to how students make and express meaning about specific mathematical content (Baxter, Woodward & Olson 2005; Cobb, 2002). Although reason and reasoning as part of most mathematical practice is often approached as interweaved with logic, proof, argumentation or even rhetoric, reasoning in this study focuses mainly on primary school students and how they embrace reasoning to explain and justify, or even express, extend or transform mathematical ideas.

In previous research, mathematical reasoning tends to focus on problem solving when examining or supporting students' mathematical reasoning, but reasoning also requires other important mathematical competences, such as explaining and discussing mathematical phenomena or expressing the meanings of ideas, concepts, operations and processes as they evolve in specific mathematical tasks. In most primary schools in Sweden, students commonly experience reasoning tasks as part of their work with problems found in the mathematics textbook, especially since individual textbook use has become a common learning practice (Boesen, Helenius, Bergqvist, Bergqvist, Lithner, Palm & Palmberg, 2014). Thus, each student is mostly expected to create his or her own understanding of mathematical concepts through comprehending tasks and solving problems as they read them in the textbook. As such, for students to be able to engage and participate in mathematical reasoning, they first need to engage with successful comprehension strategies while they read textbook-based mathematical tasks.

However, many studies of high school or university students show that such strategies are often not employed or even developed and this results in them not being able to create viable meanings of mathematical concepts, thereby negatively influencing their leaning (Shepherd, Selden & Selden, 2012; Weinberg, Wiesner, Benesh & Boester, 2012; Österholm, 2006). Similar findings were found in Segerby (2014) studies of Grade 3 and Grade 4 students. This is especially the case with Grade 4 students because it is during this year that the textbook places greater demands on students' reading skills since the text is longer and more abstract concepts are introduced compared to earlier grades (Myndigheten för skolverket, 2008). Further, a close relation between students' reading skills and reasoning competence seems to appear in Grade 4, which could indicate that the reasoning skills developed by students in early grades may accompany them throughout their school lives. Many studies, such as Baxter et al.'s (2005) and Nunes, Bryant, Evans, Gottardis and Terlektsi's (2015), stress that schools must explicitly plan to improve mathematical reasoning in direct relation to mathematical concepts and not simply teach reasoning through problem solving. But, research studies based on real classroom conditions are scarce, although some examples can be found in studies that focus on supporting students' mathematical reasoning connected to specific word problem tasks (see, for example, Collen, 2011; Huber, 2010). In the present study, mathematical reasoning refers to how students explain and describe mathematical ideas and phenomena such as the meaning of certain words or the significance of concepts, operations and processes as they evolve when mathematical problems are confronted in the context of reading textbook tasks.

In the field of reading, research involving both quantitative and qualitative studies (e.g. Lederer, 2000; Pressley, 2000) often applies Palinscar and Brown's (1984) Reciprocal Teaching (RT) approach where the strategies of *prediction, clarification, questioning* and *summarization* are emphasized to support students' reasoning when reading a text. However, the effects of implementing the Reciprocal Teaching towards supporting students reasoning in reading has been emphasized by scholars, but it has not been widely examined in other content areas such as mathematics and, at the same time, a range of appropriate teaching activities to use when implementing the RT seems limited (Rosenshine & Meister, 1994). At the same time, students' mathematical reasoning is highly interwoven with language use, specifically, in regard to how they read and how they explain in oral, visual and written forms. Language use in school mathematics has been seen through the Systemic Functional Linguistic (SFL) approach as a phenomenon that evolves in between a specific situation and the context of culture. In other words, the RT approach can contribute towards creating specific pedagogic tasks that support students' reasoning skills whilst SFL can contribute towards exploring students' language use in the context of such tasks as a matter of the specifics of the given situation and the context of culture (Halliday & Hasan, 1985).

Taking into account that, on one hand, there are few suitable activities that would create a supportive context for students' mathematical reasoning (Brehmer, Ryve & Van Steenbrugge, 2015; Thompson, 2014), and on the other hand, teachers at most times express uncertainty or the inability to deal with classroom-based reasoning (Liberg, 2008; Ratekin, Simson, Alvermann & Dishner, 1985), the present study focuses on creating a collaborative intervention study that attempts to support students' reasoning in school mathematics. As such, the collaborative intervention design coordinates basic tenets from the Reciprocal Teaching approach for identifying appropriate complementary tasks that scaffold mathematical reasoning and Systemic Functional Linguistics for exploring how Grade 4 students embrace reasoning as language use through such tasks. At the same time, the study will attempt to explore and account for the engendered potentialities and boundaries in such an endeavour. In the following sections, reasoning in primary school mathematics practice and the provided conceptual framework is discussed before providing a description of the methods used to generate the data. The article ends with a discussion of how appropriate activities can potentially scaffold students' reasoning and of how the context of culture might influence students' mathematical reasoning growth.

2.0 Reasoning in the primary school mathematics practice

Reason and reasoning have been discussed as core elements when the function and nature of mathematical practices as social, historical and philosophical phenomena are being considered (see, for example, Chronaki, 2018; Toulmin, 2003,). Reasoning is emphasized by several researchers in pedagogy, but there seems to be a lack of agreement among researchers about what mathematical reasoning refers to and how could it be operationalized in the context of teaching and learning. As a result, the question remains of how teachers and students can be supported to participate in the reasoning process through pedagogic design. Some researchers relate reasoning to processes of explaining and generalizing (Bishop, Lamp, Philipp, Whitacre, Schappelle & Lewis, 2014; Makar, 2014) others to proof and ways of proving (Hanna, 2000; Styliander, 2009) and even to manners of argumentation where specific argumentative strategies are emphasized (Toulmin, 2003). However, despite such studies, there seems to be little discussion of how reasoning could be potentially developed through pedagogy or what such pedagogical practices might consist of (Sterner, 2015).

Further, in recent curricular reforms worldwide, there is an increased emphasis on maths teachers to work towards developing students' mathematical reasoning. For example, one can see explicit attempts to include reasoning as part of mathematics curriculum reforms in countries like India (NCF, 2005), USA (NCTM, 2003) and Sweden (Skolverket, 2011). In the current national curriculum for primary school in Sweden, reasoning is considered one of the five abilities that teaching practice should essentially provide for all students as an opportunity to develop mathematical thinking (Skolverket, 2011). However, a pedagogical delineation of how reasoning could be employed in the school mathematics classroom is not clearly expressed in the syllabus for elementary school. An explicit attempt at a description can be found in the curriculum document for upper secondary school where 'reasoning competence':

> "…means being able to bring mathematical reasoning involving concepts and methods to form solutions to problems and modelling situations. Bringing reasoning also

> includes both by themselves and with others test, propose, predict, guess, question, explain, find patterns, generalize and argue" (p. 2, Skolverket, w.y, p. 2, *own translation*).

As such, reasoning focuses heavily on language use, as it emphasizes processes of explaining, justifying and articulating one's thinking in oral, visual or written forms. In related research, mathematical reasoning relies on students' mathematical reasoning to different problem-solving tasks. For example, in Rojas-Drummond and Zapata's (2005) study of 88 students in Grades 5 and 6, implementing activities where the students were asked to express, share and explain their ideas to problem solving contributed towards developing students' mathematical reasoning. Positive effects could also be found in Nunes et al.´s (2015) study of 9 to 11-year-old students' understanding of probability and their ability to reason about and solve other mathematical reasoning problems when they had the possibility to discuss their finding with their peers. Furthermore, Stein's (2008) model consisting of five practices for whole class discussions is in Larsson's (2015) educational design research inspired study supported students' reasoning competence in problem-solving tasks. Thereby, research show that collaboration between students and, as well as, between students and teacher, can be an important aspect for developing students' mathematical reasoning in problem solving.

For students to be able to reason, specific reading comprehension strategies are also needed, and a close relation between students' reading skills and mathematical reasoning competence seems to emerge in Grade 4 in Sweden (Möllehed, 2001) and appear to follow the students during their whole school time. Thus, to support comprehension strategies in school mathematics as part of reasoning seems essential, particularly in Sweden, where the dominant practice for students is individual work in mathematics textbook (Boesen et al., 2014). But several mathematics teachers express that they feel unprepared for teaching 'reasoning competences' in their mathematics classrooms as they lack adequate knowledge for practices where language must be taught along with mathematics (Liberg, 2008). In research, the implementation of reading comprehension strategies for supporting students' mathematical reasoning is limited. However, in Huber's (2010) and Quick's (2010) studies, the comprehension strategies in the RT model has been implemented successfully in supporting students' mathematical reasoning and is connected to problem solving. In Quick's (2011) study, the RT strategies were applied and aligned with Polya's four stages in problem solving (i.e. see, plan, do and check) and in Huber's study, the RT strategies were solely applied to problem-solving tasks. Thereby, previous research tends to connect mathematical reasoning to problem solving. But Baxter et al.'s (2005) and Nunes et al. (2015) stress that schools must explicitly plan to improve mathematical reasoning in relation to all basic mathematical competences such as arithmetic and conceptual understanding, not just mathematical reasoning to problem solving as reasoning is a strong predictor of students' future mathematical performance. Nevertheless, suitable activities are difficult to find (Brehemer et al., 2015; Thompson, 2014).

In this study, the RT model has been employed to support students' mathematical reasoning through creating specific tasks that support students when explaining and discussing mathematical phenomena, such as the meaning of ideas, concepts, operations and processes as they evolve in specific mathematical tasks. To further explore students' engagement in reasoning, their linguistic choices are explored by taking into account the Systemic Functional Linguistics approach. The co-ordination of RT and SFL will be explained in the section below.

3.0 Coordinating reciprocal teaching and systemic functional linguistics

In this section, we will outline how, on one side, the coordination of reciprocal teaching as an approach for developing reasoning strategies, and on the other side, systemic functional linguistics as a tool for exploring language use has contributed, first, to create a collaborative design intervention for supporting reasoning in the maths classroom, and second, to account for its effects when implemented in a mathematics classroom in an urban school in Sweden. Specifically, the RT model contributes to visualizing the students' reasoning strategies, which SFL cannot. In contrast, SFL contributes towards visualizing the students' linguistic choices in a specific situation and context of culture during the classroom intervention. In earlier studies, SFL has been used for studying written and spoken language use in mathematics classrooms (see, for example, Herbel-Eisenmann, 2007; Wagner, 2012). This conceptual framework is used not only as an analysis tool but also as a tool to develop the design of the activities in the intervention. In the following sub-sections, we describe what reciprocal teaching and systemic functional linguistics are and then we explain how their coordination has been materialized in the context of reasoning tasks in relation to number-sense, `addition and subtraction´ and geometry.

3.1 The reciprocal teaching approach

The Reciprocal Teaching approach involves the comprehensive strategies of *prediction, clarification, questioning* and *summarization*, which are considered as the baseline for reasoning (Palinscar & Brown 1984). These four comprehension strategies provide a dual function, which not only improve comprehension but also provide an opportunity for the reader to check how it occurs. Furthermore, the RT model's aim is to make students aware of the factors that influence learning and resulting in them appreciating their activity as readers (Palinscar, 2007).

The strategy of *prediction* is dependent on students' ability to predict content, which in turn, necessitates drawing and testing inferences. This strategy requires making assumptions in advance about the subject of the text based on the title, subheadings and pictures (Palinscar, 2003). At the same time, it helps students connect what they already know about a topic and the new information they are going to learn and is considered as necessary to be able to develop knowledge within a topic (Carter & Dean, 2006). The strategy of *clarification* involves explaining concepts in words and phrases in a text (Palinscar & Brown, 1984), and in this study, it concerns mathematical concepts as words or phrases that are important for the students to understand to comprehend the mathematical content. In previous research (Lundberg & Sterner, 2006; Riccomini, Smith, Hughes & Fries, 2015; Schleppegrell, 2004), the mathematics-specific vocabulary was found to be problematic for students to grasp the mathematical content. In mathematics, the implementation of a dictionary has been suggested to develop students' mathematics vocabulary (Lundberg & Sterner, 2006), but that is not enough. According to Pimm (1987) students must be able to explain mathematical terms in their own words in order to develop understanding. Furthermore, to gain a deeper learning of the concepts, the students need to be able to use words for concepts across contextual settings (Stahl & Failbanks, 1986).

In addition, the strategy of *questioning* aims to support students formulate questions about certain topics and to determine whether the information presented becomes conceived. This strategy has been proven to increase students' awareness of the main ideas in a text (Palinscar & Brown, 1984). The process of formulating questions can be an indicator of how the reader makes meaning of the information provided (Palinscar, 2003). Formulating questions may differ (Sullivan & Liburn, 2002) to include open or closed questions which can generate either one correct answer in the form of 'yes' or 'no' or a word or a number. Open questions can generate a number of alternative answers that provide students the opportunity to actively participate and clearly show how they have thought through and dealt with the question. For example, through prompting the students to reason. And finally, the strategy, *summarize*, requires students to remember and organize the important ideas in a text (Palinscar, 2003). Usually one representation, such as a mathematical word or symbol, is not enough to express meaning in mathematics because the texts are often multimodal, meaning that they involve texts wherein words are integrated with numerals, symbols, abbreviations, pictures, diagrams and graphs (Duval 2006). Duval (2006) stresses that learning occurs during the transformation of semiotic representations. Thus, one of the most important aims of teaching should be to empower students to deal decisively with different situations, and concerning mathematics, this means using and making transformations between semiotic representations. In previous research, creating concept or mind maps has been employed to summarize, which also is used in this study. The map (see Figure 1) can be seen as a learning apparatus that incorporates the traditional mental tools of words, numbers, lists, lines and sequences with an additional set of mental tools that are especially powerful for memory and creative thinking (Buzan, 1991). Two previous studies in mathematics education (Brinkmann, 2003; Budd, 2004), have reported the positive effects of concept or mind maps as a creative knowledge organiser and thereby visualize their development. However, these studies were conducted with teachers (Brinkmann, 2003) and students at the university level (Budd, 2004) whilst in the present study we are focused towards supporting primary school students.

3.2 The systemic functional linguistics approach to language use

Within SFL, two contexts are considered important for language use and meaning making – the context of situation and the context of culture – and these influence each other when creating meaning about a content (Halliday, 1993; Halliday & Hasan, 1985). *Context of culture* refers to what goes on outside language – the happenings and conditions of the world as well as the social processes involved (Halliday, 2004). It refers to the beliefs, lifestyles and value systems of a language community and involves certain assumptions and expectations (Halliday, 2007). In this study, the context of culture involves perceptions about mathematics involving the roles of the teacher, the students and the textbook. *Context of situation* refers to the linguistic choices when reasoning, which in SFL refers to the mathematical register, i.e. the structure of the language. The register involves together three metafunctions: ideational, interpersonal and textual around which language use is being structured (Halliday & Hasan, 1985).

The *ideational metafunction* addresses experiences and is constituted by the *field*, which refers to what is happening, and for this the naming of objects relevant for the context is central (Halliday & Hasan, 1985). Another part of *field* concerns how actions are expressed through the process of making 'meaning' of a specific action. The most common processes are material, relational and mental (Halliday, 1973). *Material processes* involve physical actions such as those involved when counting or adding. *Relational processes* emphasize relations between objects, as for example in 'multiplication is repeated addition', and *mental processes* involve the experience of a phenomenon, as in, "I think mathematics is fantastic". The *interpersonal metafunction* is constituted by the *tenor* and highlights the roles of the participants and the choices they have in the situation from the perspective of power and status (Halliday and Hasan 1985). Relationships can be revealed by examining the 'voice' of the text through identifying the use of imperatives, personal pronouns and modal verbs (Herbel-Eisenmann, 2007), and this approach is also adopted in this study. Imperatives command readers or listeners to do something, such as 'write', whereas personal pronouns, such as 'I' and 'you', identify the participants exemplifying their personal engagement (Morgan, 1998; Wagner, 2012), and finally, modal verbs indicate the level of certainty, for example, 'shall' and 'can'. The third metafunction, the textual metafunction is constituted by the *mode* which involves what role language plays involving different representations such as symbols, words and illustrations and how they work together to create cohesiveness when expressing meaning (Halliday & Hasan, 1985).

3.3 Bringing together reciprocal teaching and systemic functional linguistics

While SFL has been extensively used as a way to understand language use in the mathematics classroom, we feel that the issue of helping students engage and participate in reasoning activities demands more than simply using the correct words; it necessitates their engagement into certain routines and norms that support the need to reason and justify action, and as such, they need to be enclosed into coherent wholes of activity. By coordinating SFL and the RT model, the analyses of the structure of the language can be made concrete within a specific culture and situation. There are also similarities between the two frameworks. Both consider that language use is integral to, and not separate from or prior to, cognitive development, and in addition, cognitive as well as sociocultural aspects are considered essential when students learn new vocabulary. Furthermore, both build on the, sometimes problematic idea that the teachers should always be the experts to help students that are always considered novices. Within the RT model, this is referred to as *scaffolding*. In this study, scaffolding involves not only developing the students' mathematical register when reasoning by examining and implementing suitable activities connected to the Reciprocal Teaching comprehension strategies and SFL, but also seeing how the teacher can scaffold reasoning as expressed by students themselves. In Appendix 1, a table shows how the RT comprehension strategies of prediction, clarification, questioning and summarization and SFL metafunctions are being placed together and, as such, have been translated into complementary tasks for scaffolding mathematical reasoning in three mathematical settings: number sense, `addition and subtraction´ and geometry in this study.

3.4 Collaborative intervention: RT and SFL to support reasoning

During a 15 week-long period, the reading comprehension strategies of *prediction, clarification, questioning* and *summarization* were implemented with the teacher one at a time and was aimed at supporting students' reasoning competence. They were used to approach three topics: number sense, addition-subtraction and geometry. During these 15 weeks, the students' everyday mathematics textbook was utilised as the material basis for the intervention. Due to the lack of appropriate reasoning tasks in the textbook a number of complementary tasks were created to support students reasoning competence. To frame the intervention design, field-based studies were conducted (such as, Ebbelind & Segerby, 2015; Segerby, 2014; Segerby, 2016) which examined students' opportunities for reasoning in school mathematics. Further, in Ebbelind and Segerby (2015), potential issues when reading the mathematics textbook for reasoning were identified. In Segerby (2014), a study of some of the students in the class where the intervention took place showed that all these students had developed reasoning strategies, but some were more successful than others, with the high-achieving students having developed more successful strategies than their peers. Nevertheless, all students still needed to develop their ways of reasoning in relation to mathematical ideas and concepts.

Initially, the teacher and Cecilia implemented the reading comprehension strategy of *clarification* to support students' reasoning competence when writing. This involved students being able to understand and describe mathematical words but also to use them towards explaining a variety of solutions to mathematical tasks. Initially, a dictionary and whole-class discussions were introduced for students. In collaboration with the teacher a number of mathematical concepts that engender some conceptual difficulty for students such as the mathematical ideas of 'digit', 'number', 'unit digits', 'tens digits', and 'hundreds digits' and tried to ask questions that refer to SFL's ideational metafunction or, in other words, what is the contextual relevance of these words. The teacher and the students jointly defined the chosen mathematical concepts on the whiteboard, and dictionary entries coming out from this exercise were handed out to the students as homework, but when explaining how to solve different tasks, the mathematics-specific words from the dictionary did not appear. The strategy of *clarification* became revised during the following weeks except for the continuing building up of the dictionary with new concepts (number line, size arranging and number pattern). In addition, exercises were provided and the teacher scaffolded the students by visualizing how different mathematical concepts could be clarified by using several semiotic resources, which from SFL's perspective refers to the textual metafunction. Several researchers (Gibbons, 2009; Minsono & Takeda, 2012) stress that scaffolding is an effective approach for supporting students' language competence in various subjects. For example, the teacher showed how to both size-arrange four numbers by placing the numbers in the correct order and how an explanatory text can contribute to making the reasoning visible. This involved developing students' textual metafunction by showing them how to use several semiotic resources. Additionally, complementary exercises were provided where the students could elaborate on adding odd and even numbers and draw conclusions from these results. The topic of 'number sense' ended with students using the comprehension strategy of *summarization* for the first time, where the students were to identify (ideational metafunction) and explain (SFL's textual metafunction) the main ideas.

In relation to the topic of 'addition and subtraction', the students continued working with clarification and the strategy of *prediction* was introduced to develop students' skills for identifying main ideas. This was an issue for the students when summarizing the topic of 'number sense' after working with the area. Specific guidance and instructions by the teacher were provided in both oral and written text, for example, reading the headings and reading the information boxes to identify the main ideas, which refers to 'naming', which is found in SFL's ideational metafunction. The teacher explained how this strategy becomes essential for students' knowledge development as past and new knowledge can be connected and initially the students wrote down something they know about addition and subtraction. In the area of 'addition and subtraction', *clarification* was modified to also involve the students' clarifying solutions. Specific tasks connected to the concepts in the dictionary not only involved the implementation of mathematical functions and symbols such as addition, subtraction or the equal sign but also tasks where the students were required to provide descriptions and explanations to solutions in different tasks, which refers to SFL's ideational and textual metafunctions. In these tasks, we changed the receiver of the students' notes to be a fictive young child, a grade 3 students instead of the teacher, to determine if that could make students' reasoning more visible and cohesive concerning all SFL's metafunctions: ideational, interpersonal and textual. After working with addition and subtraction, students were to use the strategy of *summarization* for the second time to *identify main ideas* (SFL's ideational metafunction) and *explain* them (SFL's textual metafunction). This step allowed us to study if there was any progression in students' reasoning. Furthermore, during the teaching of this topic, several tasks involving developing the students' procedural knowledge in addition and subtraction were also used complementary as the majority of students needed to develop such skills.

In geometry, students worked with all four comprehension strategies during a five-week period. Initially, the students were asked to write down (summarize) what they know about geometry before starting to work with the area (i.e. words relevant for the context, 'naming', which refers to SFL's ideational metafunction). The students then continued with the strategy of *prediction* during the entire geometry chapter by themselves and without guidance by writing what they thought the text was about before starting to work.

During the geometry chapter, the students were also asked to formulate a *question* after reading and working with some of textbook tasks on three occasions. In practical terms, this meant that students started by individually writing down their questions and answers which were connected to distance, various measurements, figures and perimeter, and thereafter, communicating the same in groups. Initially, several of the students did not provide a question that included the current mathematical content 'distance', being expressed on the pages. The second time the students should create a question that involved millimetres (mm), centimetres (cm), decimetres (dm) and metres (m), and the teacher explicitly said that the questions should relate to the mathematical content on the textbook pages. To extend students' questioning skills, the teacher talked about how questions can be constructed. Instead of asking questions, the teacher asked students to create a task connecting figures and perimeter and guided them that answers should not be found explicitly on the pages but they would provide an explanatory text where the imperatives of 'describing' and 'explaining' were discussed as part of interpersonal metafunction. In addition, the comprehension strategy of *clarification* was modified and added to the dictionary to answer their own questions.

At the end of the chapter, the students also worked with their peers to discuss how to clarify certain mathematical concepts such as square, rectangle, triangle and circle before engaging in the whole-class discussion where these new concepts were clarified and entered as new words in their dictionary. The geometry chapter ended up with both a test in geometry and summarization of the topic for third time. This time, when *summarizing* geometry, the teacher explained to students how these notes were important not only for visualizing one's own development in geometry, but also for remembering what they did as they had encountered the subject of geometry at next time.

4.0 Context of the study: Classroom culture, participants and methods

As previously explained, the aim of the present study is to explore how a specific collaborative intervention design that coordinates RT and SFL might support students' reasoning process and to determine the engendered potentialities and boundaries in such an endeavour. A crucial step towards this aim was setting up a close collaboration with the teacher and his classroom. The teacher was a 45-year-old male maths and science specialist with 15 years of experience. The use of the mathematics textbook dominated in this particular classroom, and students had to follow step-by-step tasks, learn theory and perform tests that actually framed how students were enculturated into mathematics – by doing tasks, solving problems and completing exercises, and the students were required to perform these procedures quickly. The students working individually in their textbook and frequent homework assignments were dominant activities that framed the mathematics culture in this classroom. Yet, the teacher was curious and interested in participating in the study to explore more of this dimension, which is required by the curriculum where reasoning is stressed as one of the five abilities a student should develop in elementary school mathematics. Reasoning has been amongst the five abilities emphasized in the Swedish curriculum; the others are performing procedures, problem solving, communicating and conceptualizing (Skolverket, 2011)

Not including the teacher, the 18 participants of this study were nine- and ten-year-old students in Grade 4. The class was at a small school located in a small town in Sweden. All students have Swedish as their first language, except for one who was of an immigrant background but was born in Sweden. Most of the students were classified as having a lower socioeconomic background (i.e. working class). The students' overall ability in mathematics varied. According to the class' previous teacher, five students are considered as high achievers, eight as middle achievers and five as low achievers based on previous tests and assessments in Grades 1–3. In the present study, 18 students agreed to participate in the intervention, but three of them handed in their notes sporadically, and thus, the number was reduced to 15. Although, all students participated in the study during the entire period, the present paper will bring in data only from four students of different skills in reading and writing and varying levels of motivation and ability in maths as well as gender. It is hoped that through providing an overall picture of how the class worked as a whole, and at the same time discussing how these four students have worked within the classroom culture will provide deeper insights of how an intervention that aims to support students' reasoning can be implemented in the context of a mathematics classroom in Sweden and help to explore engendered complexities. All four students are 10 years old: two boys and two girls. Of the boys, August was considered by the teacher to be a high achiever in both mathematics and reading, while Ben was a middle achiever in mathematics who had some problems with writing notes because of his weak sensorimotor skills, which affected his handwriting. Of the two girls, Amy was considered a high achiever in mathematics but a low achiever in reading, and Eve a girl who was considered a low achiever in mathematics.

Data collection was based on participant observation of all lessons, and involved keeping observation notes in a word and collecting students' work on the tasks designed to support students' reasoning during a 15 weeks period. The students' notes were collected, copied and compiled ones or twice times each of the weeks and observation notes were kept during all of the mathematics lessons. As mentioned, the tasks were co-designed with the teacher and incorporated basic tenets from both the Reciprocal Teaching and SFL approaches. Observation notes contributed to explore and identify features of the classroom culture that were distinctive of the norms and rituals performed by the teacher, the students and the tasks.

Data has been triangulated by relying on lesson observation diary notes and students' notes. Cecilia has also discussed her interpretation of findings with the classroom teachers continuously and fellow researchers at several times. The analysis aimed to identify the potential (and the boundaries) of the designed collaborative intervention towards supporting students to participate in reasoning about 'number sense', 'addition' and 'geometry'. This type of collaborative design research has been discussed in the context of Educational Design Research studies, where Gravemeijer and Cobb (2006) argue how collaboration between researchers and practitioners becomes a required bond. The researcher is interested in studying the process of teaching in cyclical periods where practice and theory interact as cycles (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). The teachers are often interested in changing something in their practice, which, in this case, refers to developing students' participation in reasoning in school mathematics. In the following section, the students' participation will be presented and discussed analytically in the three mathematical settings we worked on with the teacher and the students: 'number sense', 'addition and subtraction' and 'geometry'.

5.0 Number sense – An entry to reasoning or into unknown waters?

Number sense and, specifically, a focus on clarifying and summarizing main ideas in relation to number (odd and even numbers, digit, place value, units, tenths, hundreds and thousands) provided an entry to reasoning during a five-week period of lessons. *Clarifying* and *summarizing* are two of the basic tenets suggested by the reciprocal teaching approach, and they were worked through practices such as whole-class discussions, dictionary creation and use as well as complementary tasks where students were asked to describe and explain in their own ways. Initially, a whole-class discussion was held with the teacher to discuss the children's previous experiences with numbers where it became apparent how difficult it was for them to give their own descriptions or explanations. They could not remember, concentrate or even articulate their thoughts orally or in writing. Most children in the class had difficulty engaging in this practice, and some even expressed frustration at not having enough time to work in the mathematics textbook. Our urge to initiate them in tasks that support reasoning was indeed an entry into unknown waters. This shows how strong the culture of textbook-based practice still is in mathematics classrooms in Sweden, and at the same time, the absence of any recognition of the need to move towards a culture of reasoning, even in the form of encouraging students to provide simple descriptive explanations or express their own ideas.

In response to children's difficulties, we turned to work more with them the process of concept clarification through the task of an off-hand dictionary co-created with the classroom teacher and students. It was due to the co-creation of this dictionary that their entry into reasoning was negotiated and became the basis on which they had to make descriptions and explanations. Realizing that this was a new practice for them, we asked them to continue as homework to allow them more time to think and practice with fuller descriptions and explanations. The homework specifically required the students to think over two tasks: The first was related to clarifying a concept – 'digit' – and the second was related to place value through explaining how they solved the task – 'What is the biggest number you can create with using the digits 6, 5, 8 and 2 and explain how you arrived at this number'. Most of the children responded to the first question 'What is a digit?' with symbols but not words. Specifically, our four students wrote down all the digits either from 0 to 10 (i.e. August and Eve) or from 1 to 10 (i.e. Ben) or else referred to the set of the 10 digits as a whole (i.e. Amy). The second task, 'Create the biggest number', seemed easy for most of children, and they responded correctly, but their reasoning was limited with few mathematics-specific words such as 'unit digits' and 'hundred digits', which were included as part of their dictionary. Our four students responded correctly and provided their explanations based on prior knowledge of 'place value'. For example, August explained, "8652 –För att 8000 är störst sen kommer 600 sen 50 sen 2" (because 8000 is the highest, and then comes 600, then 50, then 2). Amy stated, "8652 – Högsta siffran är först om man gör så får man det hötsta talet”(Highest digits first if you do like that you get the highest number”, and Ben explained, “8652 – Jag tar den största först och fortsätter sen i storleksordning”(I take the highest first and continuing by size arranging”, and finally, Eve stated “8652 –För att 9 är störst av 6,5,2, så jag tänkte att 8 först 6 sen 5 sen 2 sen”(Because 8 is the highest of 6, 5, 2 so I thought 8 first 6 then 5 then 2 then”.

The students’ work was discussed as part of a whole-class discussion where the teacher tried to emphasize the variety of their practices that can lead to *clarifying* a concept. At this initial stage, the aim of clarifying concepts was met with resistance from some of the students because they said this took longer to perform than the tasks the students were used to solving and demanded more writing. Thus, reasoning to clarify concepts requires not only time, but also a re-organization of the classroom culture in terms of tasks and activity in relation to the students’ and teacher’s role. As a consequence, some complementary tasks were devised where the students had to get involved into clarifying new concepts such as size arranging and number line. At this stage, the students started to provide more of their own examples by making visual illustrations of what a number line is or by providing an explanatory text to clarify the new concepts, as the four students’ notes exemplify in Table 1.

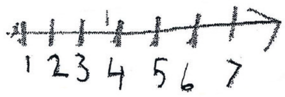
August:	
Amy:	“Det är en linje med tal” (It is a line with numbers)
Ben:	
Eve:	“Du sätter ut så att man har siffor med” (You put out so you have digits with)

Table 1. Students’ explanatory notes for the question ‘What is a number line?’

Our work in ‘number sense’ ended with practising tasks focusing on *summarization*, where the students were asked to recap what they learned about number sense. This strategy required students to identify and organize important ideas when reading a text (Palinscar, 2003). In our study, the students had difficulties comprehending that summarizing can be part of their work in mathematics despite having worked with the main ideas of ‘number’ when building up a dictionary and working with complementary tasks such as ‘reread the text’ and ‘recount or verify what have done in the task’. As a result, creating a mind map on the board as a whole-class discussion was introduced; where, the teacher identified the core concept and the class, in collaboration with the teacher, contributed with interconnected and expanding ideas.

How could we interpret the students' entry into reasoning in relation to clarification and summarizing as basic tenets of the RT approach and through the SFL perspective? Initially, when the students were asked to *clarify* the concept of 'number digit', they imitated small parts of what was written about digits in the dictionary, usually involving symbols. This is referred to as the textual metafunction in the SFL perspective, and it involves the semiotic resources that appear, such as digits and/or words, and how they contribute (either together or individually) to the cohesiveness of the text (i.e. create meaning). Also, none of the students mentioned the number values of unit, ten, hundred or thousand digits, when clarifying 'digit' or in their explanatory text when explaining how they constructed the highest number, which is referred as the ideational metafunction by SFL and means naming relevant for the context (Halliday & Hasan, 1985).

However, giving students vocabulary lists with definitions, as being recommended by Lundberg and Sterner (2006), was shown to be not enough for them to develop conceptual meaning in relation to the words used. The students initially only imitate what was written in the dictionary without referring to the number values in their solutions. Just as Pimm (1987) stresses, students need opportunities to practice and explain mathematical words and concepts in their own personal ways in order to foster deeper understanding of the concepts. In our study, this was also found when students were asked to clarify the concept of 'number line' after they discussed with the teacher what a number line is and added this concept to their dictionaries. It provided students with opportunities to use their own words or examples when clarifying concepts, which is stressed by Pimm (1987) as important because students gain mental ownership of concepts via their own language use. However, the result showed that none of the students' notes clearly defined what the concept of 'number line' involves. All four students referred to 'number line' as 'numbers' or 'digits' or 'line' in written text or as an illustration, except for Eve who did not mention 'line'. In the illustrations, an arrow is included in the number line and the distance between the lines are correct. From an SFL perspective, these aspects refer to the ideational and textual metafunctions. Specifically, *ideational* refers to the main ideas being expressed (e.g. number line), while the textual metafunction refers to how the students use semiotic resources to explain number line and how they contribute to show how the students had made meaning (e.g. if they have both provided an illustration of a number line and/or an explanatory text). Nevertheless, Eve and Amy provided an explanatory text in words, but the words did not clearly express what a number line is. Thus, none of the students' clearly expressed the concept of 'number line' because none of them provided an example and explanatory text along with the example, which made it difficult to determine if they had understood the concept or not.

When *summarizing* number sense, only Amy suggested concepts relating to number sense word 'number'. This indicate that the other three students were not able to identify or transform the concepts (main ideas) from the dictionary into a larger context, which refers to SFL's ideational metafunction. This indicates that the students had not developed a deeper understanding of the concepts, as they were not able to define the concepts in any appropriate way or transform the concepts into another context (Stahl & Failbanks, 1986), except for Amy who was able to do this with one of the concepts. Also during this topic, the students showed resistance to reasoning because the students were not familiar with the activities, the teacher's role and their role. Thereby, their assumptions about mathematics –the context of culture– did not fall in line with the new approach. Thus, based on the above, we may be able to argue that students' entry to reasoning was a delicate entry to unknown waters.

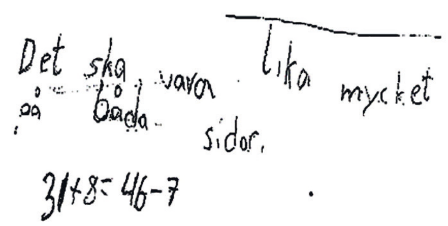
We could also argue that, overall, at this initial stage of implementing the intervention, the four students had similar reasoning skills. In terms of the strategies of clarification and summarization, the students copied and imitated the dictionary and the teacher the first time they were asked to use the strategy. Further, the second time the students were required to clarify certain concepts, the students began to use either their own words or own examples, but not both, which also shows their reasoning was limited. However, it is worth noting that Amy, who is considered a high achiever, was the only student of the four who was able to transform the mathematical concept of 'number' into another context, which is essential for gaining a deeper understanding of a concept. The ability of high-achieving students to use more successful strategies than the other students for reasoning was also found in one of the field-based studies (see Segerby, 2014).

To move forward, the activities needed to be revised to support students to use mathematical concepts (the main ideas) in other contexts, which in SFL, refers to the ideational metafunction. Further, when clarifying mathematical concepts and solutions, students' notes should involve several semiotic resources to make their reasoning richer and cohesive; this refers to SFL's textual metafunction.

6.0 Addition – Reasoning as creating 'maps' for a 'third' other

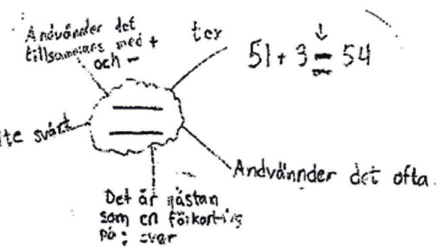
As far as the operation of 'addition' is concerned, the children were specifically focused on using the Reciprocal Teaching strategies of clarifying, summarizing and predicting in relation to the process of adding and the meaning of the equal sign among the broader concepts of 'addition' and 'subtraction' during a five-week period. By means of a whole-class discussion and in collaboration with the teacher, the students had to keep notes about the meaning of specific words, such as the act of adding as well as the signs that symbolize addition and equality. They then had to clarify their meaning to contribute towards creating a dictionary of mathematical terms. It was noted that students had enormous difficulty in engaging and participating in tasks that required them to think both abstractly and reflexively in relation to concepts. The main difficulty was that they could not see the point in such an endeavour. In response to this, the strategy of clarification was modified by introducing the ‘third other’ in the task. Specifically, the students were asked to imagine that the receiver of their notes was a younger student (i.e. Grade 3), and given this, they should make their reasoning as rich and as clear as they could. It was interesting that the imaginary presence of a third other facilitated almost all of the students in the class to describe mathematical concepts and operations and involved both their own example(s) and explanatory text, as can be seen in the following figures, see Figure 1 and 2. Although during the whole-class discussions, the students’ reasoning in relation to both addition and subtraction was worked through, here we will focus the analysis mainly on students’ reasoning in addition and numerical equality including the equal sign. Figure 1 specifically shows students’ reasoning maps about the equal sign, whereas

August:



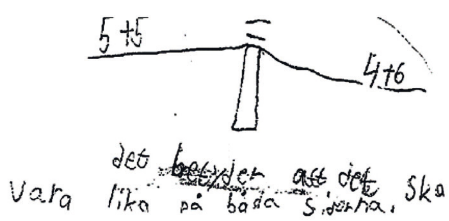
(It shall be equal on both sides)
31 + 8 = 46 – 7)

Amy:



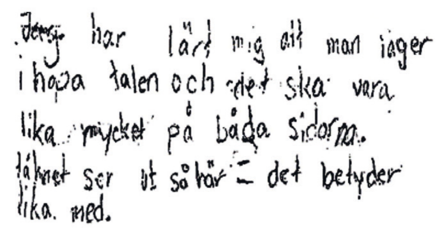
(Use it together with + and –, for example, 51+3=54, use it often, it is a shortening of answer, not difficult)

Ben:



(It means that it shall be equal on both sides)

Eve:



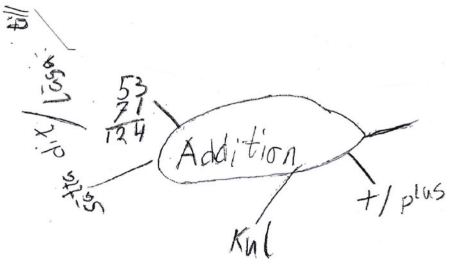
(I have learnt that you should add numbers and that is shall be equal on both sides. The equal sign looks like this=it means equal)

Figure 1. Children explain what the equal sign is.

In terms of the equal sign, which is a basic notion when addition and subtraction are being considered, Figure 2 represents how our four students attempt to clarify and summarize what they understand about ‘equal’. This refers to naming relevant to understanding the equals sign’s function as also the SFL’s ideational metafunction denotes. In all students' notes the phrase "equal on both sides" was mentioned, except for Amy who wrote instead, "almost a shortening of the answer" and might be taken as a misunderstanding of the equals sign's function. August, Amy and Ben's notes involved both explanatory text and examples, such as '31+8 = 46 – 7', which contributed towards exemplifying students' understanding and misunderstanding about the mathematical concept of the equal sign in a clearer way and comes close to textual metafunction through the SFL's perspective. Ben showed a different understanding by providing an illustration of the equals sign's function by depicting a wave. However, Eve only wrote an explanatory text without providing specific examples. Just as she previously used clarification to explain number line; again, this led her to less clarity about her own understanding of the equal sign function and her reasoning in direct relation to a specific mathematical concept seem not to have developed much. In the class, the general pattern was to provide both example and explanatory text.

Reasoning about the operation of addition involves a direct engagement with the engendered algorithm taught very explicitly in the Swedish classroom. Furthermore, as part of concluding what they experienced in varied tasks on addition (i.e. algorithmic of addition with higher numbers, such as 2326 + 2867) students were asked to explain what is addition by creating a mind map where they should summarize what they now know about addition after working with the topic. Examples from the four students' maps are presented in Figure 2.

August:

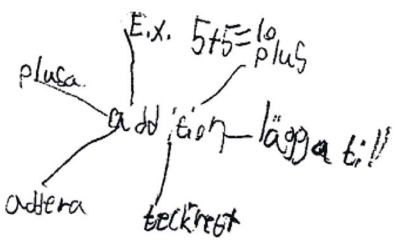


(plus, fun, put together)

Amy:

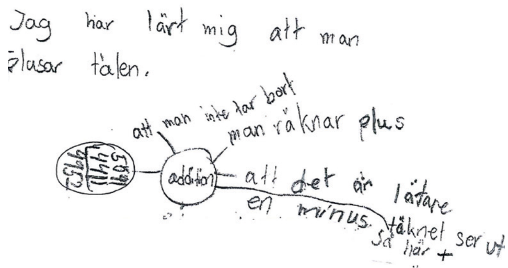
(not difficult, looks like this, use it in almost all we do, you use it when you take away numbers)

Ben:



(plus, put together, sign, add, plus)

Eve:



(I have learnt that you add numbers, not take away, you count plus, easier than minus, the sign looks like this+)

Figure 2. Students' summarization of 'addition'

When *summarizing* a topic for the second time, all of the students were able to identify the main ideas, such as 'add', 'put together' and 'provide own examples of addition', which almost none of the students in the class were able to do in 'number sense'. In students' notes, both their own example(s) of addition and words that described what to do (material processes) such as 'add', 'put together', and the addition sign (+) appeared. At this point, the students' maps began to organize students' knowledge, which Brinkmann (2003) and Budd (2002) also found in their studies of older students. For this to happen in this study, the students needed to work to develop their mathematical reasoning connected to the strategies of *prediction and clarification*. Overall, students' notes involved single words and examples. However, Amy provided a sentence, "Use it when you take away numbers", which probably refers to one of her examples "45 +__ = 63". Again, her notes contributed to show her misunderstanding, which could not be shown in her operations. At this point feelings appeared in Eve's, Amy's and August's notes that involved the experience of addition. Amy wrote "not difficult", and Eve wrote "that is easier than minus" and August expressed "fun".

In this topic, the strategy of prediction was introduced, where the students were asked to *predict* about the main ideas on different pages in this topic of 'addition and subtraction' before they started to work on different pages. This strategy is essential for activating prior knowledge, which had previously been presented to be an issue for several students according to the results from one of the field-based studies (Segerby, 2014) in the class because they were not able to predict (i.e. identify the main ideas on different pages). In the previous topic, 'number sense', almost none of the students could identify main ideas. Initially, the teacher provided explicit instructions for how to predict about the content by using headings and the information given in the information boxes and, in collaboration with the students, they identified the main ideas of 'addition', 'addition sign' and 'equal sign'. This aspect refers to SFL's ideational metafunction because it concerns naming relevant for the context (Halliday & Hasan, 1985). The teacher also explained why it is important to use this strategy. Thereby, language use was given a function in the development of the students' mathematical reasoning. Also, the students were made aware of the importance of language use in school mathematics for their learning and other parts in the text. It became apparent that not just the exercises (which previously was the main focus) need to be taken into account when reasoning and this involved a shift towards changing the classroom culture.

To summarize, we saw that, after having worked in this topic, the students' mathematical reasoning seemed to be developed in all three strategies of prediction, clarification and summarization. The students were able to use and identify mathematical concepts relevant for the context, such as 'unit digits', when they solved different operations, and when summarizing topics, such as 'add', which differed from the topic of 'number sense', where only Amy was able to transform a mathematical concept into another context when summarizing. Mental processes appeared in August, Amy and Eve's notes, which related to their personal experience and reflected in writing such as "not difficult". All of these aspects refer to SFL's ideational metafunction. In terms of clarification, August and Ben showed a deeper understanding with their provided examples; August has written in his notes '31 + 8 = 46 – 7' in contrast to Amy, who wrote '51 + 3 = 54', which showed that the total amount needed to be taken into account on both sides of the equals sign. Here, when working with this strategy, Amy's misunderstanding is evident – even though she performed the operations correctly, she did not reflect the correct meaning about equals sign's function. Also, all the students used several semiotic resources when clarifying and summarizing 'addition'. However, Eve and Amy's summarization was much richer and involved longer texts, whereas August and Ben's notes mainly contained single words. Further, Amy and Eve used higher numbers in their operations, while August and Ben provided numbers under 100. Nevertheless, none of the students turned towards connecting explanatory text to their operations which they had previously done when asked to clarify the operation. Again, it seems that the students were still working along the lines of "do it fast, and do it quick", and summarizing was difficult for them to appreciate perhaps because it places higher demand on students' language use, which again, is a process that the students are not performing when they engage with tasks in the mathematics textbook (i.e. the context of classroom culture). From these results, the students' mathematical reasoning needed to be connected to the strategy of summarization.

7.0 Geometry – Is there a need for all students to reason?

In the topic of geometry, students worked with all four strategies for the first time during a five-week period, and involved the content of 'distance' measuring units (i.e. mm, cm, dm and m) and 'figures' (i.e. circle, square, rectangle and triangle, as well as perimeter). The whole-class discussions continued, as well as the complementary exercises, and in addition, group and peer discussions were formed. As an entry to work in this topic, the students were asked to write down what they knew about geometry as a way to determine their prior knowledge of the topic. Their explanations can be seen in Table 2.

August:	"Former" (Forms) 
Amy:	"Jag vet nästan ingenting om geometri" (I know almost nothing about geometry)
Ben:	"Former och sådant" (Forms and such)
Eve:	"Jag vet inte" (I do not know)

Table 2. The students' answers to the question, "What is geometry?"

However, the purpose of this task was not only to determine students' prior knowledge of geometry but also to see how they have developed this knowledge. This means that a comparison between the students' prior knowledge and their knowledge after working with geometry could reveal some sort of change that could exemplify development.

In this topic, the strategy of *questioning* was introduced, and students were asked to provide questions about the mathematical content after working with different pages in the mathematics textbook; here, *clarification* was modified to answer their own questions. During the topic of geometry, all students were able to *predict* about the mathematical content (i.e. identify main ideas such as perimeter and figures) on their pages without guidance from the teacher.

In the beginning, the teacher needed to explicitly tell students that the questions should involve the mathematical content, which several of the students' questions did not involve initially even though they were able to predict about the main idea distance. The second area measurement. All students made the correct assumptions when predicting about decimetre (dm), centimetre (cm) and millimetre (mm). August, Amy and Eve predicted "dm and mm" and Ben "dm", and their questions involved the mathematic content. Some examples from the four students' provided questions after working on these pages about geometry and reflect upon this can be found in Table 3.

Questions	Replies
"Hur långt är 2 m I dm" (How long is 2 m in dm?)	"20" by August
"Hur lång är en tändsticka?" (How long is a match?)	"4 cm" by Amy
"Hur långt är 1 dm?" (How long is 1 dm?)	"10 cm" by Ben
"Är 10 cm I dm?" (Is 10 cm 1 dm?)	"Yes" by Eve

Table 3. Students' questions about different measurements.

All students' questions were closed, involved one correct answer (Sullivan & Liburn, 2002), and started with the interrogatives "How long?" (August, Amy and Ben) or "Is" (Eva), which in SFL's perspective, refers to the interpersonal metafunction. This construction is similar to a variety of tasks in the mathematics textbook (Ebbelind & Segerby, 2015, Segerby, 2014). The students' answers (*clarification*) to the questions involved a short answer, a number, or a 'yes' or 'no' that could be found in the information box on the pages, which refers to SFL's textual metafunction. Thereby, the students imitated what was written in the textbook. The teacher discussed with the students the importance of being able to describe and explain their thinking and instead of asking a question the students were asked to provide a task next time. However, the students' tasks involved limited reasoning even though the imperatives describe and explain started to appear but usually only involved clarifying concepts, such as a square.

At the end of working with geometry, the students were asked to summarize what they know by creating concept maps. The teacher also highlighted that their maps as presented in Figure 3 could work as memory maps next time they encounter geometry. Upon completion of their maps, students compared their notes and figurations and had an opportunity to visualise knowledge development. The four students' maps can be found in Figure 3.

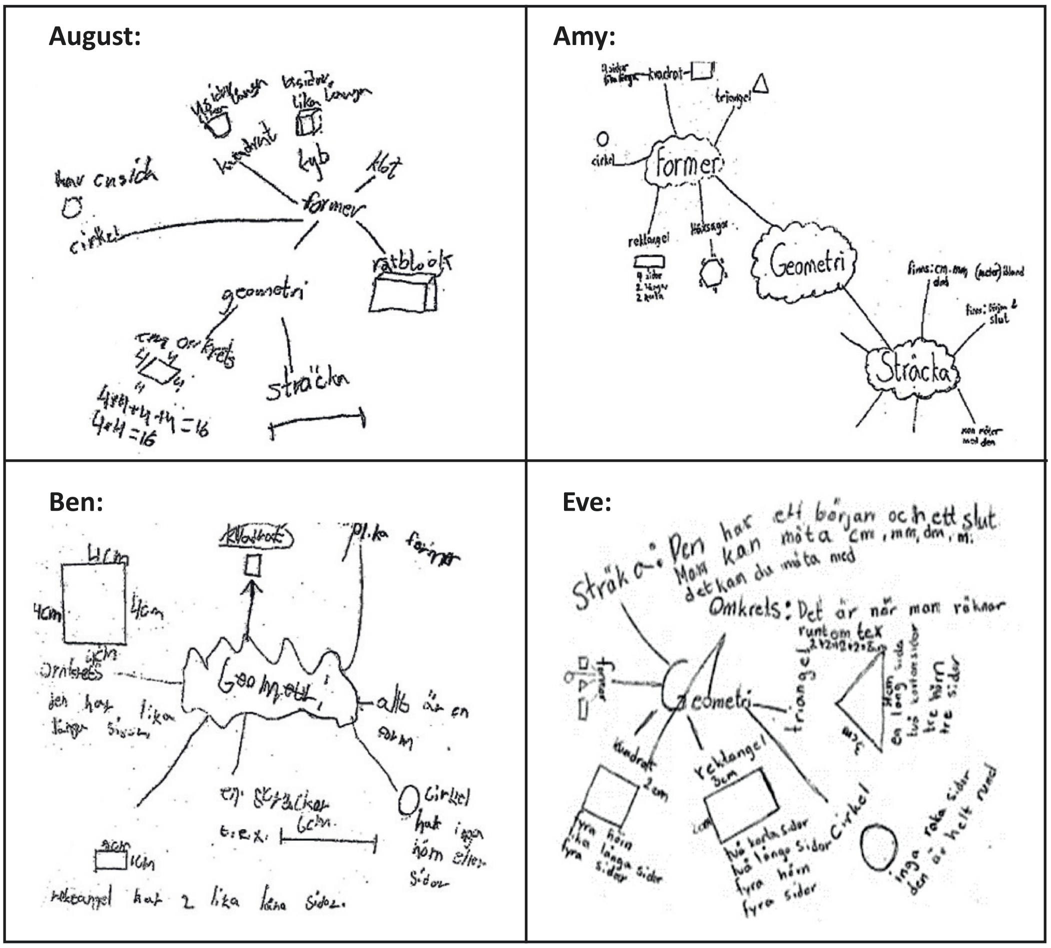


Figure 3. The students' mind maps of geometry.

Initially, Amy and Eve believed they did not know very much about geometry and provided no specific information before working with the area geometry. August provided illustrations of some geometric figures, and Ben and August wrote the word, 'forms'. After working with the topic connected to the figures, mathematics-specific words appeared in all students' notes, for example, 'rectangle'. Additionally, Eve provided an explanatory text, which accompanied the figures in all concepts. August and Amy also explained in their texts the relation between sides (sidor) and corners (hörn) in rectangles and squares. Also, the word, 'perimeter (omkrets)', appeared in August's, Eve's and

Ben's notes, and Eve and August also described how to find the perimeter of an area: Eve used the material process of 'count around (räkna runt)' and August used the mathematical operations of '4 + 4 + 4 + 4 = 12 cm' and '4 x 4 = 12'. Here, they used the mathematical concept of 'perimeter' interwoven with how the subject and the materiality involved working together as a system. 'Distance' (sträcka) was found in all four students' notes. Amy and Eve described *distance* as something that can be measured (material process) and/or has a beginning and an end (relational process). All of these aspects refer to SFL's ideational metafunction. Furthermore, illustrations, mathematics-specific words and explanatory text relating to the different figures were provided by all of the students; for example, Ben and August depicted 'distance' with an illustration. Thus, several semiotic resources that contribute to cohesiveness appeared in all the students' notes, which refers to SFL's textual metafunction, and it is here where the students' understanding and misunderstanding can be seen. Later, the students compared their prior knowledge and current knowledge after working with the subject, and many students were very pleased about the results because they could see how their knowledge of geometry had developed. At this point, all the students were able to transform the concepts into another context when constructing tasks, answering their tasks, and when summarizing the topic. Thereby, the students' mathematical reasoning had been developed positive. Further, the students also became more active as participants, where both group- and peer-discussions became a part of the teaching, yet the teacher's scaffolding remained essential.

8.0 Discussion

The aim of the present study was to explore how a specific collaborative intervention design that coordinates RT and SFL may support Grade 4 students' reasoning process and what might be the engendered potentialities and boundaries in such an endeavour. In previous research, reasoning has been stressed as important for mathematics learning by several researchers and in varied national curricula. For example, in Sweden, where this specific intervention took place, reasoning is currently being discussed as one of the five most essential abilities that the teaching and learning of mathematics should aim. However, what the act of reasoning could involve and which type of activities or tasks may be used to support students' mathematical reasoning has been explored mainly connected to problem solving. However, as stressed by other researchers (Baxter et al, 2005; Nunes et al., 2015), basic mathematical competences such as conceptual understanding and arithmetic also need to be taken into account in the process of reasoning. In this study, we aimed to suggest suitable activities based on RT that could support directly students' mathematical reasoning as part of explaining and discussing mathematical phenomena such as the meaning of ideas, concepts, operations and processes as they evolve in specific mathematical tasks. In the present study, we aimed for a collaborative intervention assuming that, for the students to be able to reason about mathematical content as part of text-based tasks, they need to employ specific comprehension strategies because reading in school mathematics differs from reading texts in other subjects. By using the Educational Design Research (EDR) approach, various activities were implemented and refined during the 15-week intervention in a Grade 4 class to support their mathematical reasoning, and three phases were identified. The preceding analysis enables us to discuss students' potentials for mathematical reasoning in relation to a number of issues, first, in relation to the varied phases of the collaborative intervention, second, as part of students' varying abilities and skills in reading and writing or mathematics, and third, in direct relation to the mathematics classroom culture in Sweden.

8.1 Reasoning phases as part of the collaborative intervention

During our collaborative intervention, three reasoning phases could be identified as part of the intervention, which characterize students' participation in reasoning and can be called 'imitative', 'limited' and 'richer' reasoning. During the first phase that we could call imitated reasoning, one could observe that all students' notes connected to all strategies, and this was where the students could copy the dictionary or textbook or else imitate the teacher. For example, Ben formulated the question, "How long is 1 dm?" but this information was explicitly expressed in the information box of the text and led to the assumption that he might have copied the information from the textbook. According to Barton and Heidema (2002) and Pimm (1987), this is an issue, as the students cannot obtain mental ownership of the terms when imitating somebody or copying something. The present study also has found that students were not always able to transfer the current concepts into another context (Stahl & Failbanks, 1986), except for Amy. However, imitation was adequate as far as the strategy of *prediction* was concerned where the teacher could give clear instructions in operationalising this strategy successfully, such as reading the heading and all information boxes to identify the main ideas. This is situated in the SFL ideational metafunction as it involves naming relevant for the context. Specifically, this would involve the use of single words, such as 'perimeter' and at the beginning, the teacher argued about its importance for mathematical reasoning.

The second phase could be denoted as 'limited reasoning' and involved students working with the strategies of *clarification, questioning* and *summarization*. Here, the teachers' scaffolding was essential, and the students received support for reasoning and other important skills, such as the use of several semiotic resources like providing examples and explanatory text (SFL's textual metafunction) were highlighted and made mathematical knowledge more accessible (Schleppegrell, 2004). At this point, the students could identify the main idea when summarizing, which refers to SFL's ideational metafunction concerning naming relevant for the context. The students also began to use their own words and/or examples to mathematical concepts when clarifying and summarizing, but not both, which in SFL's perspective, refers to the textual metafunction and concerns the use of semiotic resources. As a result, the students' understanding could not be clearly shown or identified.

In phase three, the students' mathematical reasoning become richer, and this appears as part of working with the strategies of *clarification* and *summarization*. At this point, the students were able to transform mathematics-specific words, such as 'unit digits' and 'tens digits' from the dictionary into their solutions, their questions (tasks) and also when summarizing, which is essential for understanding the concepts on a deeper level (Stahl & Failbanks, 1986). From an SFL perspective, naming relevant for the context. Further, several semiotic resources to the mathematical concept and solutions appeared; according to Duval (2006), using several semiotic resources are important because learning occurs when a transformation of semiotic representations happens. At this point, the students' notes contributed in showing students' understanding and misunderstanding in a much clearer way than before, which several researchers (Baxter et al., 2005; Cobb 2002) stress that mathematics reasoning can contribute with. However, for this to happen, the strategies needed to be modified. With *clarification*, the receiver of the note, a fictional Grade 3 became essential for developing the students' reasoning, *questioning* strategy was modified to constructing a task, and finally, *summarization* for showing their development within a topic.

8.2 Students and reasoning phases

By employing the RT strategies of prediction, clarification, questioning and summarization, the students became aware of what reasoning is about or what the process of reasoning might involve, and by scaffolding, they also began to develop a shared sense of language use in being able to reason. SFL provided more awareness of language use and supported a path for building up the required mathematical register for reasoning. However, the strategies needed to be modified and why and how to use them also needed to be clearly explained through the three reasoning phases as identified above. The result showed that students' reasoning had been developed by making reasoning explicit for all the students, but in different ways.

With August, a high achiever in mathematics and reading, his clarification and summarization strategies developed to where the use of mathematical concepts were interwoven with how the subject and the materiality involved working together as a system. However, although his summarization skills were developed, he still used single words and his explanatory texts were limited. Perhaps this is another case of "Do it fast, and do it right" thinking, which may reflect that he may not value this new approach. Amy, another high-achieving student in mathematics but low achiever in reading, specifically developed her clarification skills, and in her notes, mathematical concepts were interwoven with how the subject and the materiality involved working together as a system. Further, the strategy of *clarification* contributed to show how she had misunderstood the content which might not have been shown if she had not been asked to reason. This may not been an issue before, but in Grade 4, the text becomes more complex and thereby puts higher demands on the students' reading skills for reasoning, which in this case, could have become a later issue if this had not been discovered in her notes.

Ben, a middle-achiever in mathematics, developed his questioning and clarification skills in particular. He also developed his summarization ability, but single words frequently appeared and he offered no explanatory text, which may be difficult for him because of his handwriting issues. Eve, a low achiever in mathematics, developed her clarification, summarization and questioning skills in particular, where the use of mathematical concepts was interwoven with how the subject and the materiality involved working together as a system. Initially, it took longer for her to develop her reasoning, but when reasoning became more and more explicit by the teachers' scaffolding method and after taking into account the discussions with her peers and the teacher, she was the student who developed her reasoning the most. Moreover, she was the student who initially used unsuccessful strategies when reasoning. This indicate that before the intervention took place the students who could participate in the process of reasoning was the students that had found successful strategies by themselves, such as how to predict about a content. However, when implicit aspects became explicit for all of the students during the intervention increased the students' participation in reasoning.

The strategy that facilitated all students' reasoning competence involved *clarification*, which according to the field-based study, was an issue faced by all students wholly independent of their achievement level (Segerby, 2014). Initially, the students found it difficult to explain and describe mathematics concepts and explain their solutions to different tasks. This strategy seemed important to grasp for being able to ask questions and summarize because it is included in both these strategies.

8.3 Students, phases of reasoning and classroom culture

However, when examining the development of the students' reasoning competence, the context of culture also needs to be taken into consideration because it influences the outcome, which will be further discussed in next section. In the beginning, the students struggled to appreciate the new tasks involving reading and the new activities during mathematics lessons because they were used to being expected to perform exercises quickly, and these exercises usually involved procedures and the answers were usually short. This is consistent with many of the exercises provided in their mathematics textbook (Ebbelind & Segerby, 2015; Segerby, 2014). This is an issue because in Sweden using the textbook is considered the main way to learn mathematics according to students, their parents and the teachers; therefore, learning is a matter of textbook reading where few opportunities for the students to reason is offered (Brehemer et al., 2015; Thompson, 2014).

Initially, the students did not appreciate the whole-class discussions where mathematical concepts and solutions were discussed because the students were used to working individually in their mathematics textbooks, which a common practice in other classrooms in Sweden as well (Boesen et al. 2014). However, during the intervention, the whole-class discussion and peer discussion became essential for the development of the students' mathematical reasoning. Similar findings can be found in previous research (Nunes et al., 2015; Rojas-Drummond & Zapata, 2005) studies. The teacher's role also changed from helping students with tasks to also explicitly showing structures for reasoning when reading and writing texts (i.e. 'scaffolding' the students). Thereby, the students could gain access and a common language for mathematical reasoning (Schleppegrell, 2007). The result of the present study – that there is a need to offer students different opportunities and an environment rich in tasks which invite students to use language in varied ways; this can take the form of exploratory discussions, visual language, and written tasks in the form of maps or even text to develop their mathematical reasoning. This took time because huge changes in the classroom culture were needed. This may not be the case in other classrooms because classroom cultures are likely to be different in other classrooms and in other countries. However, what the result of this study highlights is that there is a need for teachers and researchers to not isolate mathematical reasoning away from the considerations and contexts of culture and contexts of situation that operate in those classrooms and countries.

8.4 Concluding remarks

Reasoning is seen as a strong predictor of students' learning in mathematics, but previous research tends to focus on reasoning connected to problem solving. But also, other mathematical competences need to be taken into consideration, such as conceptual understanding. An essential aspect of this study is that one of the most important aims of teaching should be to empower students to deal decisively with different situations; in terms of mathematics, this means using and making transformations between semiotic representations. This might suggest that teaching about both how to read and write in mathematics is highly relevant for helping students interpret instructions and information in school mathematics in relation to being able to reason including all mathematical competences not just problem solving. Implicit aspects about reasoning need to be made explicit. This should not be something tricky for the students to find out by themselves, because it can result in more successful or else less successful strategies that can follow the students during their whole school lives. Therefore, teaching and guiding students (scaffolding) about the language of mathematics is essential and should start in the earlier grades because it is through language that mathematics is taught and it is through language that students' understanding of mathematical concepts is shown and evaluated in school. As Halliday

(1993) stresses, “Language is the essential condition of knowing, the process by which experience becomes knowledge” (p.94).

Yet, this is a small-scale study where reasoning is limited to the four comprehension strategies as suggested by RT and through SFL’s approach on language use. Future research is suggested where this initial testing of the resulting design is a basis for future iterations of developing students reasoning skill in mathematics by examining the activities further in other contexts.

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